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RECENT PROGRESS IN SOLAR FURNACES FOR HIGH-TEMPERATURE RESEARCH AND DEVELOPMENT WORK

BY

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ABSTRACT

Principal advantages and disadvantages of solar furnaces in comparison to other means of obtaining high temperatures are discussed. The first large solar furnace units were used for qualitative work in which it was essential to heat small samples rapidly to maximum temperatures under very pure conditions. A melt was obtained and collected in a crucible. Solar furnaces for quantitative work have been recently developed for determining, for example, liquidus curves in binary systems. Equipment for heating and cooling a specimen according to a pre-determined schedule, for maintaining it at a desired temperature (limited by the length of the day), and for air-quenching it is described. The determination of the temperature of the sample independently of the condition of its surface is discussed in which blackbody conditions are approximated. A compilation and evaluation of factors entering into the design of experimental solar furnaces is presented. The development of solar furnaces for semi-industrial use is discussed in which the one-piece mirror has been replaced by a collector consisting of a large number of small curved mirrors.

I. INTRODUCTION

In a searchlight, an arc in the focus of a concave mirror emits parallel light into space. In the solar furnace, parallel rays from the sun are concentrated in a small area near the focus of the mirror. The sample or target is placed in the image of the sun and heated by incident radiation under controlled conditions. Both qualitative and quantitative measurements can be accomplished in a modern solar furnace while the sample is being heated or heat-treated.

The average apparent surface temperature of the sun is considered to amount to 6000° K. Temperatures attainable in a solar furnace will be less than this value, due to the losses of energy during passage through the atmosphere, the limited aperture of the collector, the losses

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by absorption and reflection of incident radiation in the optical system, etc. A computation by Dr. J. Farber of San Diego, Calif., gave 4000° K. as the maximum temperature attainable for a flat target and 4200° K. for a hemispherical target if a solar furnace 120 in. in diameter with a 34-in. focal length is used.

Principal advantages of a solar furnace in comparison to other means of obtaining very high temperatures are: (1) heating samples by solar energy presents a means of carrying work out under very pure conditions in an atmosphere which remains oxidizing up to the highest temperatures; (2) no interference occurs from electric or magnetic fields, from products of combustion or from heated furnace walls made from materials different to the sample being tested; (3) specimens may be heated or heat-treated in predetermined cycles; and (4) physical changes occurring in a sample may be directly observed up to the highest temperatures while the material is heating or cooling. The following are a few of the disadvantages inherent in solar furnaces: (1) furnaces heated by gas, oil, coal or electric energy can be heated for twenty-four hours while solar furnaces can only be used during daylight hours; (2) solar heating is dependent on favorable atmospheric conditions; and (3) the area of heating is limited in size depending on the focal length of the main mirror and the definition of the sun's image.

If the tests require melting of the sample, the molten material may be collected in a refractory crucible made from or lined with the same material that is being studied. Many materials must be heated above their melting points in order to decrease the viscosity and surface tension of the melt and to bring about a steady flow of molten material into a crucible where it will form, upon cooling, crystalline or glassy products.

This paper presents a discussion of and design data for modern solar furnaces used in experimental work. A recently built semi-industrial solar furnace will be briefly mentioned.

II. MEASUREMENT OF TEMPERATURE

The dimensions and optical characteristics of the mirror which collects the radiation incident from the sun will determine the temperature attained per unit area of the sample which is brought into the image of the sun. An equilibrium temperature is attained which depends on the amount of radiant energy incident on and absorbed by the sample, and on the energy lost through emission, convection and conduction.

The temperature of a specimen heated below its melting point can be measured by an optical pyrometer or a radiation pyrometer on a surface opposite or perpendicular to the incident radiation if the sample is small enough to attain uniform temperature. The temperature of a larger sample or target must be measured from the same side from which it receives radiant energy. The result is that the incident radiation from the sun reflected by the concave mirror and the temperature

radiation of the sample are superimposed. They must be separated in order to obtain the brightness temperature of the sample or target. Difficulties arise in obtaining the true temperature of a material of rough or uneven surface and of substances with unknown emissivities at high temperatures.

The following steps were taken to permit accurate control and measurement of the temperature of a sample:

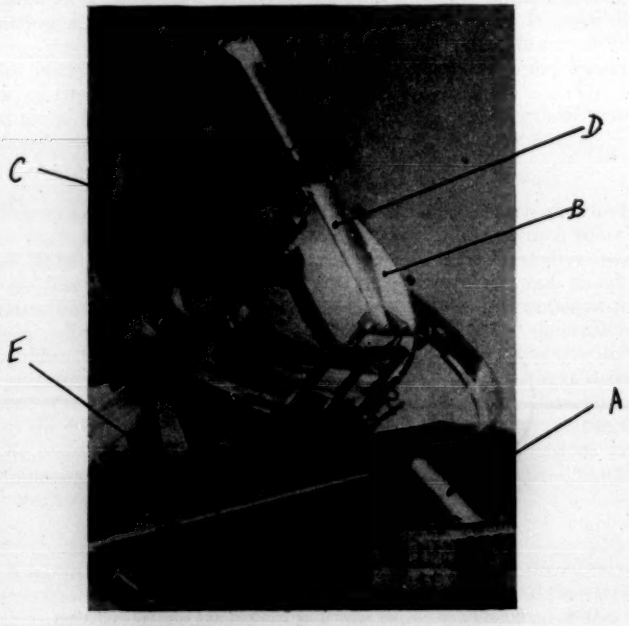

FIG. 1. Solar furnace atop Rockhurst College, Kansas City, Mo., equipped with 120-in. aluminum reflector and aluminum cylinder for control of temperature.

A—Polar axis (equatorial mounting)
B—120-in. mirror
C—Aluminum cylinder for control of temperature by varying the effective aperture of the mirror
D—Bridge supporting sample holder
E—Lever arm for rotating polar axis connected to astronomical controller

(1) An aluminum cylinder traveling along the optical axis of the mirror was installed. It permits control of the temperature of the sample by admitting varying amounts of solar radiation to the sample. This is equivalent to a variation of the effective aperture of the mirror. Figure 1 shows the use of the cylinder for a 120-in. solar furnace. The

temperature of the sample can be controlled within $\pm 1°$ C. up to about 2000° C., and within the accuracy of the optical pyrometer at higher temperatures.

(2) Rotating sectors have been developed which permit the separation of the sun's incident radiation from the temperature radiation of the sample (see Fig. 2). The axis of rotation of the sectors is parallel

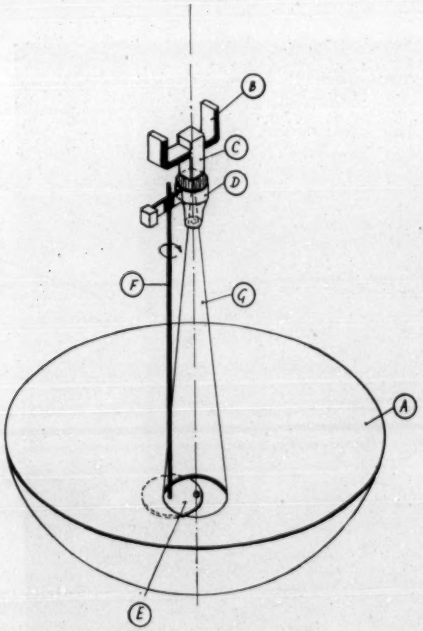

FIG. 2. Rotating sectors for separating incident and emitted radiations.

A—Concave mirror with central opening
B—Jaws of sample holder
C—Sample
D—Rotating sector near sample with counterweight
E—Rotating sector near optical pyrometer
F—Common shaft for both sectors
G—Shadow cone caused by central opening of concave mirror

to the axis of the mirror. The first sector close to the sample, of elongated spherical shape, shields the sample from incident radiation in the position shown. A small opening (stop) in the first sector coincides with the optical axis of the mirror while another stop in the second sector of disk shape, close to the telescope of the optical pyrometer, is also in the optical axis. The apparent brightness temperature of the

sample can now be measured. The sample is next exposed to incident radiation from the sun upon rotation of the sectors while the field of view of the optical pyrometer is blocked off. The sectors rotate at a rate of 2500 rpm. in order to avoid flicker for the observer.

(3) If the emissivity of the sample is not known at high temperatures, in particular near the melting point, a deep hole or crater of small diameter is drilled or burned into the sample, for example, 4 mm. in diameter and 40 mm. deep. A sample in the shape of a thin rod is mounted in a refractory enclosure equipped with a small opening for admitting radiant energy. These arrangements make it possible to measure temperatures under conditions which approximate grey or black bodies of cylindrical or conical shape. Temperatures are measured inside of the crater or the opening of the refractory cover. The temperature of the sample is maintained for the desired length of time by manipulating the cylinder for temperature control, if necessary.

III. FIELDS OF STUDY TO BENEFIT FROM SOLAR FURNACE WORK

Experimental solar furnaces have been used in research and development work at high temperatures. Fields of study in which solar furnaces have been utilized or are expected to be of particular interest include, for example, the high-temperature chemistry and physics of silicates, borides, carbides and nitrides, the thermal properties of materials in solid or gaseous form at high temperatures, the determination of liquidus curves, the production of glasses with high refractive indices, the growing of single crystals, and the development of heat-resistant alloys. Other fields of high-temperature research are the determination up to the highest temperatures attainable of the heat flux at the focus of the concave mirror, the rate of transfer of heat for various mirror combinations, an approach to the study of thermo-nuclear reactions, and spectrographic work.

IV. DATA FOR MODERN SOLAR FURNACES

A summary of characteristic data for modern solar furnaces which have actually been used for high-temperature research and development work is presented in the accompanying table. The data of the table are arranged according to apertures of the main (concave) mirrors and location of the units. The solar furnaces in Jena were designed and operated by R. Straubel and coworkers of the Zeiss Company in Jena, Germany; the furnaces in Meudon and Mont Louis, France, by Trombe and coworkers; and the solar furnaces at Kansas City, Mo., by the author and coworkers (the largest Kansas City unit was recently transferred to San Diego). The bibliography at the end of this paper gives some of the principal publications. Solar furnaces with less than 60-in. apertures have been in use, in particular for biological, chemical and engineering research in which low or medium ranges of temperature were desired.

Examples of Modern Solar Furnaces.

Column No. 1	2	3	4	5	6	7	8	9	10	11
Location	Kansas City I	Kansas City II	Jena I*	Jena II	Jena III	Meudon	Mont Louis I	Kansas City III	Kansas City IV***	Mont Louis II
One-piece mirror	x	x	x	x	x	x	x	x	x	
Composite mirror								Three Zones		3500 Plates
Material of main mirror	Cu	Cu	Glass	Glass	Glass	Glass	Glass	Al	Al	Glass
Reflecting material	Rh	Rh	Ag	Ag	Ag	Ag	Ag	Al	Al	Ag
Reflecting surface	Ist	Ist	IInd	IInd	IInd	IInd	IInd	Ist	Ist	IInd
Aperture of main mirror	60.0″	60.0″	69.0″	78.7″	78.7″	78.7″	78.7″	120.0″	120.0″	35.0 ft.
Focal length of main mirror	25.75″	26.0″	35.4″	33.9″	33.9″	33.5″	34.0″	34.0″	34.0″	19.7 ft.
Focal ratio of main mirror.....f/	0.43	0.43	0.51	0.43	0.43	0.43	0.43	0.28	0.28	0.56
Diameter of sun's image (computed) in.	0.24	0.24	0.33	0.32	0.32	0.31	0.31	3×0.32	0.32	2.20
mm.	6.1	6.2	8.4	8.0	8.0	7.9	7.9	3×8.1	8.1	55.8
Diameter of central opening in main mirror, in.	—	8.3	—	12.0	7.9	—	—	22	22	66**

* Private communication.

** Estimated.

*** Now in San Diego, Calif.

Examples of Modern Solar Furnaces.—Continued.

Column No. 1	2	3	4	5	6	7	8	9	10	11
Auxiliary mirror:										
Heliostat										
Heliostat size					98.4″ dia.					43 ft. × 34 ft.
Heliostat distance from main mirror					33.1 ft.					80 ft.
Newtonian			x			x	x			
Newtonian diameter						17.7″	17.7″			
Newtonian distance from focus						3.9″	3.9″			
Cassegrainian				x						
Mounting:										
Altazimuth	x					x	x			
Equatorial		x	x	x				x	x	
Stationary					x					x
Guiding:										
Manual	x					x	x	x		
Bimetal/resistor			x	x	x					
Phototubes										x
Astronomical controller		x							x	
Vacuum attachment		x		x	x				x	
Temperature control:										
Shades or diaphragm	x			x	x					
Aluminum cylinder		x							x	
Temperature measurement	x	x	x	x	x	x	x	x	x	x
Rotating sectors		x							x	

Mirrors used in solar furnace work are fabricated from glass or metal. In most glass mirrors, known also as "mirror lenses," the second (back) surface is silvered. Preliminary tests of paraboloidal glass mirrors with aluminized first (front) face gave little chemical and physical aging, thus making their use in solar furnaces practical because of the reduction of losses by absorption and reflection in comparison to mirror lenses.

Apertures of the solar furnaces of the table range from 60 to 120 in. for one-piece mirrors (consisting of a single piece of glass or metal). The aperture of the mirror of a solar furnace should depend on the program of research to be carried out, the location of the equipment, and the difficulties in procuring or manufacturing large mirrors. It is not necessary, for example, to obtain a 100-in. mirror if the work contemplated can be accomplished by a 60-in. mirror.

The *focal length* of the mirror determines the diameter of the image of the sun (near the focal area of the mirror) into which the sample or target is brought for heating. A decrease of focal length for a given aperture will result in a decrease of the diameter of the image of the sun and in an increase of the temperature per unit area of the sample. An increase in focal length will result in a larger solar image and a more extended zone of constant temperature so essential in some work, but also in a decrease of temperature per unit area of the sample.

The *central opening* in some of the concave mirrors of the table serves for machining and grinding the mirror, observing the sample during heating, reading the temperature by means of the optical pyrometer, manipulating the crucible for collecting molten materials and, in case of Cassegrainian mounting (see below), for bringing the incident and reflected radiation to the outside of the main mirror.

Auxiliary mirrors are used in heliostats or Newtonian and Cassegrainian systems (see Fig. 3). The plane heliostat mirror is set up to follow the apparent path of the sun. It reflects the incident radiation to the main concave (spherical or paraboloidal) mirror which is stationary or moves very little during a day's run of the furnace. The Newtonian type mirror is a flat mirror which is placed between the collector of energy and the sample to be heated. It intercepts the converging rays and directs them to the location of the sample which becomes more accessible or more easily adjustable for the work at hand (for example, the testing of materials in powdered form). A hyperboloidal convex mirror placed between the paraboloidal and the sample is used in the Cassegrainian type of furnace. The incident radiation is intercepted, made parallel and brought to a focus outside of the concave mirror by means of a lens. The equivalent focal length of the system can be considerably increased and most of the losses by aberration eliminated.

A stable *mounting* of the solar furnace which is free of flexure and

vibration is of particular importance for large furnace units. The main mirror of the furnace or its heliostat must closely follow the apparent path of the sun. This may be accomplished either by mounting the furnace in an altazimuth mounting in which adjustments in altitude and azimuth are made, or by mounting it in an equatorial mounting in

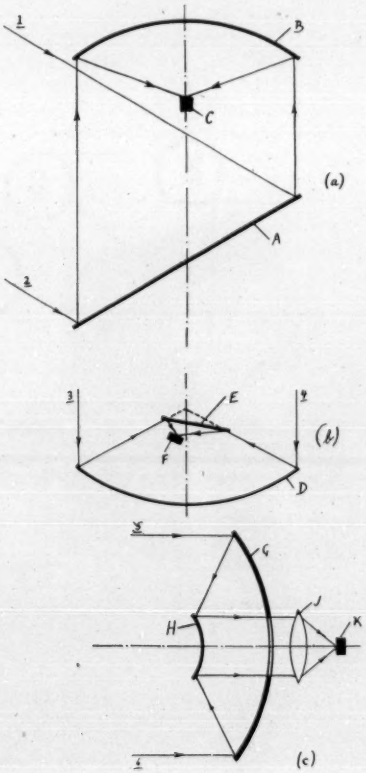

FIG. 3. Auxiliary mirrors.

(*a*) Heliostat: *A*—Plane mirror; *B*—Paraboloidal mirror; *C*—Sample; 1 and 2, incident radiation.

(*b*) Newtonian mounting: *D*—Paraboloidal mirror; *E*—Flat mirror, tilted; *F*—Sample; 3 and 4, incident radiation.

(*c*) Cassegrainian mounting: *G*—Paraboloidal mirror; *H*—Hyperboloidal mirror; *J*—Lens system; *K*—Sample; 5 and 6, incident radiation.

which the furnace is not changed during the day in declination. It follows the sun merely by rotation of the polar axis. Tests of short duration may be carried out without guiding the furnace. Experiments with the 60-in. and the 120-in. solar furnaces showed that manual

FIG. 4. Solar furnace mounted in Kansas City with 60-in. mirror.

A—Searchlight shell, altazimuth mounting
B—Copper mirror, rhodium coated
C—Supporting bridge

guiding is possible only for experiments up to about 60-sec. duration. All other tests should be carried out with automatic guiding of the equipment. Devices for guiding include bimetal strips, resistors sensitive to heat, phototubes and astronomical controllers.

Most of the work with solar furnaces has been carried out in oxidizing *atmosphere* up to the highest temperatures. An enclosure of the sample and its holder in an air-tight glass container is desirable for work in vacuum or at elevated pressure, with or without protecting atmosphere.

An example of a modern solar furnace for experimental work was presented in Fig. 1, which is identical with our furnace in column 10 of the table. Figure 4 shows a rebuilt 60-in. searchlight being used as a solar furnace, column 2 of the table. This unit is manually guided and serves for short-time exposures.

Another type of solar furnace is mentioned in column 11 of the table. It has been designed for *semi-industrial* work. The reflector of the heliostat and the collector of paraboloidal shape are composed of small back-silvered glass mirrors. It was found difficult to obtain a good definition of the image of the sun by bending flat glass mirrors to the shape of a paraboloid in this type of construction although the focal length amounted to 19.7 ft. It remains to be seen if similar compound mirrors can be utilized in high-temperature research. The furnace gave a rather high output at intermediate temperatures, for example 132 lb. of steel were produced per hour in pilot plant operation. The designer expects to obtain better definition and larger output with solar furnaces of increased focal length.

V. SUMMARY

The data presented show that there are now three main types of solar furnaces available for high-temperature research work: (1) large units that can be used in many fields but which are rather cumbersome and expensive pieces of equipment; (2) medium-sized units sufficient for many applications, in particular under favorable climatic conditions; and (3) units of simple construction and limited in use to tests of short duration. The trend in the development of solar furnaces is expected to be an adaptation to specific fields of research. The choice of the equipment and the technique will depend on a careful survey of all the factors entering into the problem. We anticipate that solar furnaces will find their rightful place alongside of and supplementing furnace units based on conventional fuels now in use.

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V.
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ANALYSIS OF NETWORK RESPONSE TO PERIODIC WAVES

BY

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The steady-state response of a network to a non-sinusoidal periodic wave may be found directly from the frequency and phase-response characteristics of the network, by making use of the simplified transform method described. This method is based on a paper by D. L. Waidelich (1)[2] and may be used whenever the network is linear and made up of a finite number of lumped values of resistance, capacitance, and inductance.

Figure 1(*a*) shows a four-terminal network in which E_{in} and E_{out} represent the input and output voltages, respectively.

If E_{in} is a sine wave, the vector diagram of Fig. 1(*b*) shows the

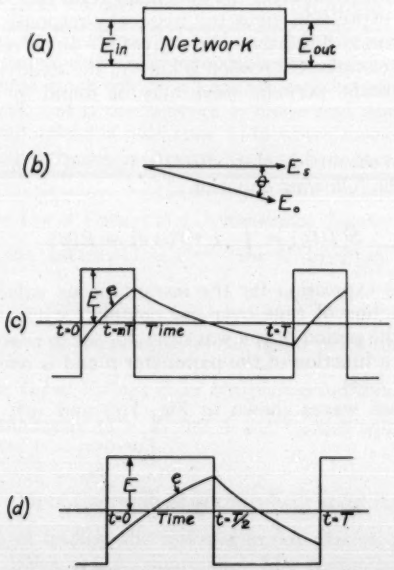

FIG. 1. Sketches illustrating the types of response being considered.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

possible relative magnitudes of the input voltage, E_s, and the output voltage, E_o, as well as the phase angle θ between them. The magnitude of E_o/E_s, plotted as a function of frequency, gives the frequency-response curve of the network. The phase angle, θ, plotted as a function of frequency, gives the phase-response curve of the network.

If E_{in} is a non-sinusoidal periodic wave, the results might be as in Fig. 1(*c*), where an unsymmetrical square wave of voltage is assumed, or as in Fig. 1(*d*), where a symmetrical square wave of voltage is assumed. In these figures E is the amplitude of the input square wave and e is the instantaneous value of the output voltage during the time interval $t = 0$ to $t = mT$. The ratio e/E, plotted as a function of time over this time interval, gives the steady-state response curve of the network to the periodic wave being applied.

OUTLINE OF OPERATIONAL METHOD USED IN THE ANALYSIS

The frequency and phase-response characteristics of a network are generally obtained simultaneously by determining a complex expression for the network which involves the imaginary term $j\omega$. The magnitude of this complex expression gives the frequency-response characteristic, while the phase angle of this expression gives the phase-response characteristic. If this complex expression is known, the steady-state response to any non-sinusoidal periodic wave may be found by the procedure outlined below.

1. Determine the steady-state direct transform of the input voltage wave by using the following equation:

$$S[f(t)] = \int_0^T \epsilon^{-pt} f(t)\, dt = E(p) \tag{1}$$

where $f(t)$ is the expression for the instantaneous value of the input voltage as a function of time over one complete cycle, from $t = 0$ to $t = T$, T being the period of the wave.

The result is a function of the parameter p and is referred to as the voltage function $E(p)$.

For the square waves shown in Fig. 1(*c*) and (*d*), $E(p)$ has the following values:

In Fig. 1(*c*),

$$E(p) = \frac{E}{p}(1 - 2\epsilon^{-\frac{1}{2}pT} + \epsilon^{-pT}). \tag{2}$$

In Fig. 1(*d*),

$$E(p) = \frac{E}{p}(1 - \epsilon^{-pT/2})^2 = \frac{EU}{p}, \tag{3}$$

where the symbol U is used hereafter to replace the factor $(1 - \epsilon^{-pT/2})^2$.

2. Substitute the parameter p for the term $j\omega$ in the complex expression for the network. The result is referred to as the network function $N(p)$.

3. Multiply $N(p)$ by $E(p)$ to obtain the response function $F(p)$.

4. Determine the steady-state inverse transform of $F(p)$ by using the following equation:

$$S^{-1}[F(p)] = \frac{1}{2\pi j}\int_W \frac{\epsilon^{pt}F(p)}{(1-\epsilon^{-pT})}\,dp. \tag{4}$$

This equation involves integration in the complex plane, and W represents a certain path of integration in this plane.

The result is an expression for the instantaneous value of the output voltage as a function of time t, and the period of the input voltage T. The output wave shape, obtained by plotting a curve over the time interval $t = 0$ to $t = T$, is the steady-state response of the network to the particular input voltage wave used.

RULES FOR OBTAINING THE INVERSE TRANSFORM

Although Eq. 4 may always be used to obtain the inverse transform, a detailed knowledge of the rules of complex integration is required for its solution. If the input voltage is a square wave, whether symmetrical or unsymmetrical, and if the network is linear and consists of a finite number of lumped values of resistance, inductance, and capacitance, the process of obtaining the inverse transform may be reduced to the following set of rules:

A. When the Input Voltage is a Symmetrical Square Wave

1. Replace the factor $(1-\epsilon^{-pT/2})^2 = U$ in $F(p)$ by the factor $\dfrac{2\epsilon^{pt}}{1+\epsilon^{pT/2}}$.

2. Find the sum of the residues of the result at all the poles of $F(p)$.

B. When the Input Voltage is an Unsymmetrical Square Wave

1. Replace the factor $(1-2\epsilon^{-mpT}+\epsilon^{-pT})$ which will appear in $F(p)$ by the factor $\dfrac{2\epsilon^{pt}[1-\epsilon^{pT(1-m)}]}{1-\epsilon^{pT}}$.

2. Find the sum of the residues of this result at all the poles of $F(p)$.

The process of finding the residues is not always easy, especially when multiple poles are encountered. The methods used are well known, however, and may be found in any standard textbook on the functions of a complex variable (2).

By following the rules outlined above, a table of inverse transforms

for symmetrical square-wave inputs has been set up, and is included as Appendix A. If the input voltage is a symmetrical square-wave, the inverse transform may be found directly from the table, provided the network is not too complicated. This table has been found to apply to many practical applications, including a complete square-wave analysis of a video amplifier compensated for both low and high frequencies (3).

REASONING BEHIND THE RULES FOR OBTAINING THE INVERSE TRANSFORM

Equation 4 for the inverse transform may be broken down into two parts, as follows:

$$S^{-1}[F(p)] = \frac{1}{2\pi j}\int_{W_1} \frac{\epsilon^{pt}F(p)}{(1-\epsilon^{-pT})}\,dp - \frac{1}{2\pi j}\int_{W_2} \frac{\epsilon^{pt}F(p)}{(1-\epsilon^{-pT})}\,dp. \quad (5)$$

The first term in (5) may be simplified when $0 < t < T$, so that (5) appears as follows:

$$S^{-1}[F(p)] = \frac{1}{2\pi j}\int_{W_1} \epsilon^{pt}F(p)\,dp - \frac{1}{2\pi j}\int_{W_2} \frac{\epsilon^{pt}F(p)}{(1-\epsilon^{-pT})}\,dp. \quad (6)$$

In these formulas W, W_1, and W_2 represent certain paths of integration in the complex plane (4).

Path W_1 is a vertical straight line drawn to the right of the imaginary axis, as shown in Fig. 2(a). All poles of both $E(p)$ and $N(p)$ lie to the

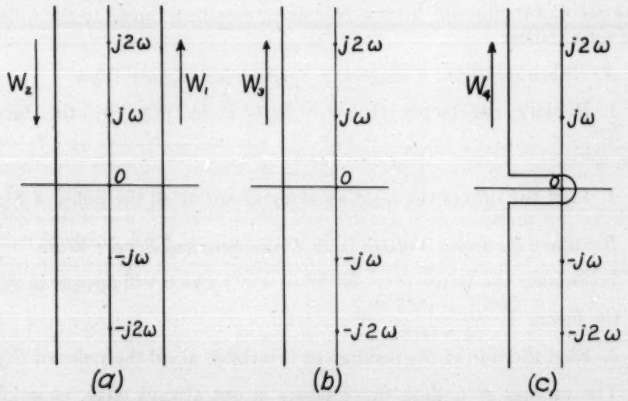

FIG. 2. Sketches showing the location of the paths of integration.

left of this path. In the case of the square wave, whether symmetrical or unsymmetrical, $p = 0$ is the only pole contributed by $E(p)$, as may be seen from both (2) and (3).

Path W_3 is a vertical straight line drawn to the left of the imaginary axis, as shown in Fig. 2(*b*). All the poles of $N(p)$ must lie to the left of this path.

Path W consists of the two vertical straight lines W_1 and W_2 shown in Fig. 2(*a*). Path W_2 is the same as path W_3, except that the direction is reversed. All the poles of $E(p)$ must lie between W_1 and W_2, but the poles of $N(p)$ are outside this region.

The method used to obtain the inverse transform from (6) can best be explained by considering a typical example. Assume

$$N(p) = 1/(p + a).$$

When this is multiplied by the direct transform of a symmetrical square wave of unit amplitude, as given by (3),

$$F(p) = \frac{(1 - \epsilon^{-pT/2})^2}{p(p + a)}.$$

When this is put in the first term of (6), the product

$$\epsilon^{pt}(1 - \epsilon^{-pT/2})^2 = \epsilon^{pt} - 2\epsilon^{p[t-(T/2)]} + \epsilon^{p(t-T)}$$

is obtained, and when $0 < t < T/2$, the effect of the second and third terms of this product is zero. This means in effect that the factor $(1 - \epsilon^{-pT/2})^2$ may be dropped from the first term of (6). The final expression is as follows:

$$S^{-1}[F(p)] = \frac{1}{2\pi j}\int_{W_1} \frac{\epsilon^{pt}}{p(p+a)}\,dp - \frac{1}{2\pi j}\int_{W_3} \frac{\epsilon^{pt}}{p(p+a)} \times \frac{(1-\epsilon^{-pT/2})^2}{(1-\epsilon^{-pT})}\,dp$$

$$= \frac{1}{2\pi j}\int_{W_1} \frac{\epsilon^{pt}}{p(p+a)}\,dp - \frac{1}{2\pi j}\int_{W_3} \frac{\epsilon^{pt}}{p(p+a)} \times \frac{1-\epsilon^{-pT/2}}{1+\epsilon^{-pT/2}}\,dp. \qquad (7)$$

The first integral is obtained by finding the sum of the residues at the poles $p = 0$ and $p = -a$. The second integral is obtained by finding the residue at the pole $p = -a$, since the pole $p = 0$ is outside the path of integration W_3.

The final result may be obtained by combining the terms of (7) before finding the residues. Although the pole $p = 0$ is outside the path W_3, nevertheless, if it is included, the result is unchanged. This is because the factor $(1 - \epsilon^{-pT/2}) = 0$ when $p = 0$, making the whole term zero. At the same time $p = 0$ is the only pole contributed by the voltage function $E(p)$. With these facts in mind, the path of integration for the combined terms is, in effect, a new path W_4 which is the same as W_3 except that it includes the origin. See Fig. 2(*c*).

Before the terms of (7) are combined, the following very useful identity is presented:

$$1 - \frac{1 - \epsilon^{-pT/2}}{1 + \epsilon^{-pT/2}} = \frac{2\epsilon^{-pT/2}}{1 + \epsilon^{-pT/2}} = \frac{2}{1 + \epsilon^{pT/2}}. \tag{8}$$

When the two terms of (7) are combined with the aid of this identity, the following is obtained:

$$S^{-1}[F(p)] = \frac{1}{2\pi j}\int_{W_s} \frac{2\epsilon^{pt}}{p(p + a)(1 + \epsilon^{pT/2})}\,dp. \tag{9}$$

The sum of the residues at the poles $p = 0$ and $p = -a$ is then found, with the following result:

$$S^{-1}[F(p)] = \frac{1}{a}\left(1 - \frac{2\epsilon^{-at}}{1 + \epsilon^{-aT/2}}\right). \tag{10}$$

This is the same as Formula 3 in the table of inverse transforms when $b = 0$.

The above discussion leads to the rules for obtaining the inverse transform. Similar reasoning leads to the rules given for the case where the input voltage is an unsymmetrical square wave.

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APPENDIX A

STEADY-STATE INVERSE TRANSFORM TABLE FOR SQUARE-WAVE INPUTS

Note: The symbol U stands for $(1 - \epsilon^{-pT/2})^2$

	$F(p)$	$S^{-1}[F(p)] = f(t)$
1	$\dfrac{U}{p}$	1
2	$\dfrac{U}{p+a}$	$\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}$
3	$\dfrac{U}{(p+a)(p+b)}$	$\dfrac{1}{b-a}\left[\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}} - \dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\right]$
4	$\dfrac{(p+c)U}{(p+a)(p+b)}$	$\dfrac{1}{b-a}\left[\dfrac{2(c-a)\epsilon^{-at}}{1+\epsilon^{-aT/2}} - \dfrac{2(c-b)\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\right]$
5	$\dfrac{U}{(p+a)(p+b)(p+c)}$	$\dfrac{2\epsilon^{-at}}{(b-a)(c-a)(1+\epsilon^{-aT/2})} + \dfrac{2\epsilon^{-bt}}{(a-b)(c-b)(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{(a-c)(b-c)(1+\epsilon^{-cT/2})}$
6	$\dfrac{(p+d)U}{(p+a)(p+b)(p+c)}$	$\dfrac{2(d-a)\epsilon^{-at}}{(b-a)(c-a)(1+\epsilon^{-aT/2})} + \dfrac{2(d-b)\epsilon^{-bt}}{(a-b)(c-b)(1+\epsilon^{-bT/2})} + \dfrac{2(d-c)\epsilon^{-ct}}{(a-c)(b-c)(1+\epsilon^{-cT/2})}$
7	$\dfrac{(p+d)(p+e)U}{(p+a)(p+b)(p+c)}$	$\dfrac{2(d-a)(e-a)\epsilon^{-at}}{(b-a)(c-a)(1+\epsilon^{-aT/2})} + \dfrac{2(d-b)(e-b)\epsilon^{-bt}}{(a-b)(c-b)(1+\epsilon^{-bT/2})} + \dfrac{2(d-c)(e-c)\epsilon^{-ct}}{(a-c)(b-c)(1+\epsilon^{-cT/2})}$
8	$\dfrac{U}{(p+a)(p+b)(p+c)(p+d)}$	$\dfrac{2\epsilon^{-at}}{(b-a)(c-a)(d-a)(1+\epsilon^{-aT/2})} + \dfrac{2\epsilon^{-bt}}{(a-b)(c-b)(d-b)(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{(a-c)(b-c)(d-c)(1+\epsilon^{-cT/2})} + \dfrac{2\epsilon^{-dt}}{(a-d)(b-d)(c-d)(1+\epsilon^{-dT/2})}$

APPENDIX A—Continued

	$F(p)$	$S^{-1}[F(p)] = f(t)$
9	$\frac{(p+e)U}{(p+a)(p+b)(p+c)(p+d)}$	$\frac{2(e-a)\epsilon^{-at}}{(b-a)(c-a)(d-a)(1+\epsilon^{-aT/2})} + \frac{2(e-b)\epsilon^{-bt}}{(a-b)(c-b)(d-b)(1+\epsilon^{-bT/2})} + \frac{2(e-c)\epsilon^{-ct}}{(a-c)(b-c)(d-c)(1+\epsilon^{-cT/2})} + \frac{2(e-d)\epsilon^{-dt}}{(a-d)(b-d)(c-d)(1+\epsilon^{-dT/2})}$
10	$\frac{(p+e)(p+f)U}{(p+a)(p+b)(p+c)(p+d)}$	$\frac{2(e-a)(f-a)\epsilon^{-at}}{(b-a)(c-a)(d-a)(1+\epsilon^{-aT/2})} + \frac{2(e-b)(f-b)\epsilon^{-bt}}{(a-b)(c-b)(d-b)(1+\epsilon^{-bT/2})} + \frac{2(e-c)(f-c)\epsilon^{-ct}}{(a-c)(b-c)(d-c)(1+\epsilon^{-cT/2})} + \frac{2(e-d)(f-d)\epsilon^{-dt}}{(a-d)(b-d)(c-d)(1+\epsilon^{-dT/2})}$
11	$\frac{(p+e)(p+f)(p+g)U}{(p+a)(p+b)(p+c)(p+d)}$	$\frac{2(e-a)(f-a)(g-a)\epsilon^{-at}}{(b-a)(c-a)(d-a)(1+\epsilon^{-aT/2})} + \frac{2(e-b)(f-b)(g-b)\epsilon^{-bt}}{(a-b)(c-b)(d-b)(1+\epsilon^{-bT/2})} + \frac{2(e-c)(f-c)(g-c)\epsilon^{-ct}}{(a-c)(b-c)(d-c)(1+\epsilon^{-cT/2})} + \frac{2(e-d)(f-d)(g-d)\epsilon^{-dt}}{(a-d)(b-d)(c-d)(1+\epsilon^{-dT/2})}$
12	$\frac{U}{p^2}$	$t - T/4$
13	$\frac{U}{(p+a)^2}$	$\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[t - \frac{T/2}{1+\epsilon^{aT/2}}\right]$
14	$\frac{(p+b)U}{(p+a)^2}$	$\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[1 + (b-a)\left(t - \frac{T/2}{1+\epsilon^{aT/2}}\right)\right]$
15	$\frac{U}{(p+a)(p+b)^2}$	$\frac{1}{(b-a)^2}\left\{\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}} - \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[1 + (b-a)\left(t - \frac{T/2}{1+\epsilon^{bT/2}}\right)\right]\right\}$
16	$\frac{(p+c)U}{(p+a)(p+b)^2}$	$\frac{2(c-a)\epsilon^{-at}}{(b-a)^2(1+\epsilon^{-aT/2})} + \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[\frac{(a-c)}{(a-b)^2} + \frac{(c-b)}{(a-b)}\left(t - \frac{T/2}{1+\epsilon^{bT/2}}\right)\right]$

APPENDIX A—Continued

	$F(p)$	$S^{-1}[F(p)] = f(t)$
17	$\dfrac{(p+c)(p+d)U}{(p+a)(p+b)^2}$	$\dfrac{2(c-a)(d-a)\epsilon^{-at}}{(b-a)^2(1+\epsilon^{-aT/2})} + \dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[\dfrac{(a-b)(c+d-2b)-(c-b)(d-b)}{(a-b)^2} + \dfrac{(c-b)(d-b)}{(a-b)}\left(t-\dfrac{T/2}{1+\epsilon^{bT/2}}\right)\right]$
18	$\dfrac{U}{(p+a)(p+b)(p+c)^2}$	$\dfrac{2\epsilon^{-at}}{(b-a)(c-a)^2(1+\epsilon^{-aT/2})} + \dfrac{2\epsilon^{-bt}}{(a-b)(c-b)^2(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{1+\epsilon^{-cT/2}}\left[\dfrac{(2c-a-b)}{(a-c)^2(b-c)^2} + \dfrac{1}{(a-c)(b-c)}\left(t-\dfrac{T/2}{1+\epsilon^{cT/2}}\right)\right]$
19	$\dfrac{(p+d)U}{(p+a)(p+b)(p+c)^2}$	$\dfrac{2(d-a)\epsilon^{-at}}{(b-a)(c-a)^2(1+\epsilon^{-aT/2})} + \dfrac{2(d-b)\epsilon^{-bt}}{(a-b)(c-b)^2(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{1+\epsilon^{-cT/2}}\left[\dfrac{(a-c)(b-c)-(d-c)(a+b-2c)}{(a-c)^2(b-c)^2} + \dfrac{(d-c)}{(a-c)(b-c)}\left(t-\dfrac{T/2}{1+\epsilon^{cT/2}}\right)\right]$
20	$\dfrac{(p+d)(p+e)U}{(p+a)(p+b)(p+c)^2}$	$\dfrac{2(d-a)(e-a)\epsilon^{-at}}{(b-a)(c-a)^2(1+\epsilon^{-aT/2})} + \dfrac{2(d-b)(e-b)\epsilon^{-bt}}{(a-b)(c-b)^2(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{1+\epsilon^{-cT/2}}\left[\dfrac{(a-c)(b-c)(d+e-2c)-(d-c)(e-c)(a+b-2c)}{(a-c)^2(b-c)^2} + \dfrac{(d-c)(e-c)}{(a-c)(b-c)}\left(t-\dfrac{T/2}{1+\epsilon^{cT/2}}\right)\right]$
21	$\dfrac{(p+d)(p+e)(p+f)U}{(p+a)(p+b)(p+c)^2}$	$\dfrac{2(d-a)(e-a)(f-a)\epsilon^{-at}}{(b-a)(c-a)^2(1+\epsilon^{-aT/2})} + \dfrac{2(d-b)(e-b)(f-b)\epsilon^{-bt}}{(a-b)(c-b)^2(1+\epsilon^{-bT/2})} + \dfrac{2\epsilon^{-ct}}{1+\epsilon^{-cT/2}}\left[\dfrac{(d-c)(e+f-2c)+(e-c)(f-c)}{(a-c)(b-c)} - \dfrac{(d-c)(e-c)(f-c)(a+b-2c)}{(a-c)^2(b-c)^2} + \dfrac{(d-c)(e-c)(f-c)}{(a-c)(b-c)}\left(t-\dfrac{T/2}{1+\epsilon^{cT/2}}\right)\right]$

APPENDIX A—Continued

	$F(p)$	$S^{-1}[F(p)] = f(t)$
22	$\dfrac{U}{(p+a)^2(p+b)^2}$	$\dfrac{1}{(b-a)^2}\left\{\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[\dfrac{2}{(a-b)}+\left(t-\dfrac{T/2}{1+\epsilon^{aT/2}}\right)\right]+\dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[\dfrac{2}{(b-a)}+\left(t-\dfrac{T/2}{1+\epsilon^{bT/2}}\right)\right]\right\}$
23	$\dfrac{(p+c)U}{(p+a)^2(p+b)^2}$	$\dfrac{1}{(b-a)^2}\left\{\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[1-\dfrac{2(c-a)}{(b-a)}+(c-a)\left(t-\dfrac{T/2}{1+\epsilon^{aT/2}}\right)\right]+\dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[1-\dfrac{2(c-b)}{(a-b)}+(c-a)\left(t-\dfrac{T/2}{1+\epsilon^{bT/2}}\right)\right]\right\}$
24	$\dfrac{(p+c)(p+d)U}{(p+a)^2(p+b)^2}$	$\dfrac{1}{(b-a)^2}\left\{\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[(c+d-2a)-\dfrac{2(c-a)(d-a)}{(b-a)}+(c-a)(d-a)\left(t-\dfrac{T/2}{1+\epsilon^{aT/2}}\right)\right]+\dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[(c+d-2b)-\dfrac{2(c-b)(d-b)}{(a-b)}+(c-b)(d-b)\left(t-\dfrac{T/2}{1+\epsilon^{bT/2}}\right)\right]\right\}$
25	$\dfrac{(p+c)(p+d)(p+e)U}{(p+a)^2(p+b)^2}$	$\dfrac{1}{(b-a)^2}\left\{\dfrac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[(c-a)(d+e-2a)+(d-a)(e-a)-\dfrac{2(c-a)(d-a)(e-a)}{(b-a)}+(c-a)(d-a)(e-a)\left(t-\dfrac{T/2}{1+\epsilon^{aT/2}}\right)\right]+\dfrac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left[(c-b)(d+e-2b)+(d-b)(e-b)-\dfrac{2(c-b)(d-b)(e-b)}{(a-b)}+(c-b)(d-b)(e-b)\left(t-\dfrac{T/2}{1+\epsilon^{bT/2}}\right)\right]\right\}$

APPENDIX A—Continued

	$F(p)$	$S^{-1}[F(p)] = f(t)$
26	$\frac{U}{p^2}$	$\frac{t^2}{2} - \frac{tT}{4}$
27	$\frac{U}{(p+a)^2}$	$\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left[\frac{t^2}{2} - \frac{tT}{2(1+\epsilon^{aT/2})} + \frac{T^2(1-\epsilon^{aT/2})}{8(1+\epsilon^{aT/2})^2}\right]$
28	$\frac{(p+b)U}{(p+a)^2}$	$\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left\{t + \frac{(b-a)t^2}{2} - \frac{T[1+(b-a)t]}{2(1+\epsilon^{aT/2})} + \frac{T^2(b-a)(1-\epsilon^{aT/2})}{8(1+\epsilon^{aT/2})^2}\right\}$
29	$\frac{(p+b)(p+c)U}{(p+a)^2}$	$\frac{2\epsilon^{-at}}{1+\epsilon^{-aT/2}}\left\{1 + (b+c-2a)t + \frac{(b-a)(c-a)t^2}{2} - \frac{T[(b+c-2a)+(b-a)(c-a)t]}{2(1+\epsilon^{aT/2})} + \frac{T^2(b-a)(c-a)(1-\epsilon^{aT/2})}{8(1+\epsilon^{aT/2})^2}\right\}$
30	$\frac{U}{(p+a)(p+b)^2}$	$\frac{2\epsilon^{-at}}{(b-a)^2(1+\epsilon^{-aT/2})} + \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left\{\frac{1}{(a-b)^2} - \frac{t}{(a-b)^2} + \frac{t^2}{2(a-b)} + \frac{T[1-(a-b)t]}{2(a-b)^2(1+\epsilon^{bT/2})} + \frac{T^2(1-\epsilon^{bT/2})}{8(a-b)(1+\epsilon^{bT/2})^2}\right\}$
31	$\frac{(p+c)U}{(p+a)(p+b)^2}$	$\frac{2(c-a)\epsilon^{-at}}{(b-a)^2(1+\epsilon^{-aT/2})} + \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left\{\frac{(c-a)}{(a-b)^2} + \frac{(a-c)t}{(a-b)^2} + \frac{(c-b)t^2}{2(a-b)} - \frac{T[(a-c)+(c-b)(a-b)t]}{2(a-b)^2(1+\epsilon^{bT/2})} + \frac{T^2(c-b)(1-\epsilon^{bT/2})}{8(a-b)(1+\epsilon^{bT/2})^2}\right\}$
32	$\frac{(p+c)(p+d)U}{(p+a)(p+b)^2}$	$\frac{2(c-a)(d-a)\epsilon^{-at}}{(b-a)^2(1+\epsilon^{-aT/2})} + \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left\{\frac{1}{(a-b)} - \frac{(c+d-2b)}{(a-b)^2} + \frac{(c-b)(d-b)}{(a-b)^2} + \left[\frac{(c+d-2b)}{(a-b)} - \frac{(c-b)(d-b)}{(a-b)^2}\right]t + \frac{(c-b)(d-b)t^2}{2(a-b)} - \frac{T[(a-b)(c+d-2b)-(c-b)(d-b)+(c-b)(d-b)(a-b)t]}{2(a-b)^2(1+\epsilon^{bT/2})} + \frac{T^2(c-b)(d-b)(1-\epsilon^{bT/2})}{8(a-b)(1+\epsilon^{bT/2})^2}\right\}$

APPENDIX A—Continued

	$F(p)$	$S^{-1}[F(p)] = f(t)$
33	$\frac{(p+c)(p+d)(p+e)U}{(p+a)(p+b)^3}$	$\frac{2(c-a)(d-a)(e-a)\epsilon^{-at}}{(b-a)^3(1+\epsilon^{-aT/2})} + \frac{2\epsilon^{-bt}}{1+\epsilon^{-bT/2}}\left\{\frac{(c+d+e-3b)}{(a-b)}\right.$ $-\frac{(c-b)(d-b)+(e-b)(c+d-2b)}{(a-b)^2} + \frac{(c-b)(d-b)(e-b)}{(a-b)^3}$ $+\left[\frac{(c-b)(d-b)+(e-b)(c+d-2b)}{(a-b)} - \frac{(c-b)(d-b)(e-b)}{(a-b)^2}\right]t$ $+\frac{(c-b)(d-b)(e-b)t^2}{2(a-b)}$ $-\frac{T[(c-b)(d-b)(a-b)+(e-b)(c+d-2b)(a-b)-(c-b)(d-b)(e-b)]}{2(a-b)^2(1+\epsilon^{bT/2})}$ $\left.-\frac{T(c-b)(d-b)(e-b)t}{2(a-b)(1+\epsilon^{bT/2})} + \frac{T^2(c-b)(d-b)(e-b)(1-\epsilon^{bT/2})}{8(a-b)(1+\epsilon^{bT/2})^2}\right\}$
34	$\frac{U}{(p+a)^n}$	$\frac{\left\lvert\frac{d^{n-1}}{dp^{n-1}}\left(\frac{2\epsilon^{pt}}{1+\epsilon^{pT/2}}\right)\right\rvert_{p=-a}}{(n-1)!}$
35	$\frac{p^{n-1}U}{(p+a)^n}$	$\frac{\left\lvert\frac{d^{n-1}}{dp^{n-1}}\left(\frac{2\epsilon^{pt}p^{n-1}}{1+\epsilon^{pT/2}}\right)\right\rvert_{p=-a}}{(n-1)!}$

THE AIRY STRESS FUNCTION IN CURVILINEAR COORDINATES WITH APPLICATION TO THE UNIFORM FLEXURE OF A NATURALLY CURVED SPIRAL BEAM *

BY

KARL S. PISTER [1]

ABSTRACT

In problems of plane elasticity, in the absence of body forces, the stresses are derivable from a scalar function known as the Airy stress function. By expressing this relation as a tensor equation, the use of the Airy function is generalized for any plane, orthogonal coordinate system. A general expression for the compatibility equation in terms of the stress function is given. It is found that considerable simplification results if isometric coordinates are used. The geometry of the isometric, curvilinear coordinate system is determined by a conformal mapping of the rectangular, Cartesian coordinate plane upon the curvilinear coordinate plane. For a given coordinate system the components of the metric tensor are expressible in terms of the mapping function.

As an application of the preceding theory the uniform flexure of a beam whose edges are bounded by logarithmic spirals is discussed. It is found that all spiral beams may be specified by two dimensionless shape parameters. Stress distributions in a typical beam are exhibited. As limiting cases of the solution for a spiral beam, the uniform flexure of a wedge acted upon by a moment at the vertex and the uniform flexure of a sector of a circular ring are obtained.

PART I. THE AIRY STRESS FUNCTION IN CURVILINEAR COORDINATES

1. *Introduction*

The idea of solving the equations of equilibrium in plane elasticity by expressing the stresses in terms of a single function was first given by Airy (1).[2] By a problem in plane elasticity we shall understand one in which the stresses are functions of only two independent variables in the plane. Such a restriction has physical significance in two extreme cases, those of plane stress and plane strain. For constant or zero body forces, which we shall assume, the solutions are identical. Airy did not consider the differential equation satisfied by his stress function, arising if the strains in terms of the stresses are substituted in the compatibility equation of strain. This leads to the reduction of all plane problems to the integration of a single linear differential equation. The first systematic treatment of this idea seems to have been given by Michell (2). In considering the general plane problem it seems natural

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

that curvilinear coordinates could be used with facility. Accordingly, we shall first develop the Airy stress function for any plane, orthogonal coordinate system.

2. General Theory

Let (ξ, η) be rectangular coordinates in the plane. Consider a one-to-one continuous mapping of a region R of the (ξ, η) plane onto a region R' of the (x, y) plane by the equations

$$\xi = \xi(x, y), \qquad \eta = \eta(x, y). \tag{1}$$

Then, (x, y) are curvilinear coordinates in the plane, since, to each set of values of (x, y) in R' there corresponds a point in R of the (ξ, η) plane, and vice versa.[3] The lines $x =$ constant are a family of curves in the (ξ, η) plane, called the y-coordinate lines. Likewise, the lines $y =$ constant are a family of curves in the (ξ, η) plane, called the x-coordinate lines. If the two families of curves are orthogonal, the coordinates (x, y) are said to be orthogonal. The linear element of the plane in terms of the curvilinear coordinates is given by[4]

$$ds^2 = g_{ij}dx^i dx^j. \tag{2}$$

If we restrict our attention to an orthogonal system of coordinates, the metric tensor in its covariant and contravariant form is

$$g_{ij} = \begin{pmatrix} E & 0 \\ 0 & G \end{pmatrix} \qquad g^{ij} = \begin{pmatrix} E^{-1} & 0 \\ 0 & G^{-1} \end{pmatrix}. \tag{3}$$

We also have $|g| = EG$, where E and G are functions of x and y that must be chosen to satisfy the condition that the Riemann tensor for the curvilinear coordinates vanishes. This is expressed by (3)

$$\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{EG}}\frac{\partial G}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\sqrt{EG}}\frac{\partial E}{\partial y}\right) = 0. \tag{4}$$

This is a special result of Gauss' equation that occurs in the geometry of surfaces. The non-zero Christoffel symbols of the second kind are

$$\left.\begin{aligned} \left\{\begin{matrix} 1 \\ 11 \end{matrix}\right\} &= \frac{E_x}{2E} & \left\{\begin{matrix} 1 \\ 12 \end{matrix}\right\} &= \frac{E_y}{2E} & \left\{\begin{matrix} 1 \\ 22 \end{matrix}\right\} &= -\frac{G_x}{2E} \\ \left\{\begin{matrix} 2 \\ 11 \end{matrix}\right\} &= -\frac{E_y}{2G} & \left\{\begin{matrix} 2 \\ 12 \end{matrix}\right\} &= \frac{G_x}{2G} & \left\{\begin{matrix} 2 \\ 22 \end{matrix}\right\} &= \frac{G_y}{2G} \end{aligned}\right\}. \tag{5}$$

[3] For the moment, only single-valued, reversible functions are considered.

[4] It will be convenient to use the summation convention. An index repeated as a subscript and superscript is understood to be summed over the range 1, 2. Free indices may assume values 1, 2. For convenience $x^1 = x$, $x^2 = y$.

These symbols are symmetric in the lower indices. Subscripts x and y are used to denote partial differentiation. In the absence of body forces the stress tensor is derivable from a scalar function $\varphi(x, y)$ according to the equation

$$\tau_{ij} = \delta^{mn}_{jp} g_{im} g^{pr} \varphi_{,nr} \tag{6}$$

where δ^{mn}_{jp} is the generalized Kronecker delta, and the comma denotes covariant differentiation. $\varphi(x, y)$ is known as the Airy stress function, and it may be shown by direct substitution that the stresses obtained from (6) satisfy identically the equations of static equilibrium. Upon making the substitution $g_{im} = \delta_{im}$, $g^{pr} = \delta^{pr}$, (6) reduces to the well known equations in rectangular coordinates. Since the equation is true for rectangular coordinates, it is true for all coordinate systems, for it is a tensor equation. The stress vector on a curve with unit normal ν^i is $\tau_{ij}\nu^i$. Consequently, the component of the stress vector in the direction of a unit vector μ^j is $\tau_{ij}\nu^i\mu^j$. Let the unit vectors ν^i and μ^j be respectively tangent to the x and y coordinate lines. Then,

$$\left.\begin{aligned} \nu^1 &= \frac{1}{\sqrt{g_{11}}} = \frac{1}{\sqrt{E}}, \quad \nu^2 = 0 \\ \mu^1 &= 0, \quad \mu^2 = \frac{1}{\sqrt{g_{22}}} = \frac{1}{\sqrt{G}} \end{aligned}\right\}. \tag{7}$$

The physical stress components are

$$\left.\begin{aligned} \sigma_x &= \tau_{ij}\nu^i\nu^j = \frac{\tau_{11}}{E} \\ \sigma_y &= \tau_{ij}\mu^i\mu^j = \frac{\tau_{22}}{G} \\ \tau_{xy} &= \tau_{ij}\nu^i\mu^j = \frac{\tau_{12}}{\sqrt{EG}} \end{aligned}\right\}. \tag{8}$$

Making use of (6) we have the result

$$\left.\begin{aligned} \sigma_x &= \frac{1}{G}\varphi_{yy} - \frac{G_y}{2G^2}\varphi_y + \frac{G_x}{2EG}\varphi_x \\ \sigma_y &= \frac{1}{E}\varphi_{xx} - \frac{E_x}{2E^2}\varphi_x + \frac{E_y}{2EG}\varphi_y \\ \tau_{xy} &= -\frac{1}{\sqrt{EG}}\left(\varphi_{xy} - \frac{E_y}{2E}\varphi_x - \frac{G_x}{2G}\varphi_y\right) \end{aligned}\right\}. \tag{9}$$

In addition to satisfying conditions of static equilibrium in a body the

stresses must be determined in such a way that compatibility of strain will be assured. The compatibility equation in terms of stresses may be written

$$\nabla^2\theta = 0 \tag{10}$$

where

$$\theta = \sigma_x + \sigma_y \tag{11}$$

and

$$\nabla^2\theta = g^{ij}\theta_{,ij}. \tag{12}$$

Carrying out the indicated differentiation in (12) gives

$$\frac{1}{E}\theta_{xx} + \frac{1}{G}\theta_{yy} + \frac{1}{2}\left(\frac{G_x}{EG} - \frac{E_x}{E^2}\right)\theta_x + \frac{1}{2}\left(\frac{E_y}{EG} - \frac{G_y}{G^2}\right)\theta_y = 0. \tag{13}$$

It is evident that considerable simplification results if we consider only isometric coordinate systems, that is, those for which $E = G$. By making the substitution

$$Q = \log E = \log G \tag{14}$$

we can rewrite Eqs. 4, 9, 11, and 13 in more compact form. They will be, respectively,

$$Q_{xx} + Q_{yy} = 0 \tag{15}$$

$$\left.\begin{aligned} \sigma_x &= e^{-Q}(\varphi_{yy} + \tfrac{1}{2}Q_x\varphi_x - \tfrac{1}{2}Q_y\varphi_y) \\ \sigma_y &= e^{-Q}(\varphi_{xx} - \tfrac{1}{2}Q_x\varphi_x + \tfrac{1}{2}Q_y\varphi_y) \\ \tau_{xy} &= e^{-Q}(-\varphi_{xy} + \tfrac{1}{2}Q_y\varphi_x + \tfrac{1}{2}Q_x\varphi_y) \end{aligned}\right\} \tag{16}$$

$$\theta = e^{-Q}(\varphi_{xx} + \varphi_{yy}) \tag{17}$$

$$\theta_{xx} + \theta_{yy} = 0. \tag{18}$$

The solution of a particular problem in plane elasticity then consists in choosing a suitable coordinate system subject to (15), and secondly, seeking a stress function satisfying (18) and giving stresses in (16) that will be compatible with prescribed boundary conditions.

3. *Geometry of the Isometric Coordinate System*

To simplify the condition expressing compatibility of strains, (13), the coordinate system was chosen to be isometric. This simplification has certain other implications. It is known from the geometry of surfaces that if, for a given surface, one isometric coordinate system (ξ, η) is known, all other such coordinate systems (x, y) are obtainable from an equation of the form

$$\xi + i\eta = f(x + iy) \tag{19}$$

where $f(x + iy)$ is an analytic function of the complex variable

$$z = x + iy. \tag{20}$$

Accordingly, we may choose the (ξ, η) rectangular system as the known isometric coordinates. Setting

$$\zeta = \xi + i\eta, \tag{21}$$

Eq. 19 can also be written

$$\zeta = \zeta(z). \tag{22}$$

In view of (22) the lines x = constant and y = constant of the z-plane can be mapped conformally and isometrically onto the ζ-plane where they will form the curvilinear coordinate net. The following relation between the mapping function $\zeta(z)$ and the components of the metric tensor of the isometric, curvilinear coordinates, $E(x, y)$, is to be noted: From (1), (2), and (3)

$$ds^2 = d\xi^2 + d\eta^2 = E(dx^2 + dy^2)$$

for isometric coordinates (x, y). By making use of the Cauchy-Riemann equations, which follow from the choice of $\zeta(z)$ as an analytic function, it is easily shown that

$$E = \xi_x^2 + \xi_y^2 = \eta_x^2 + \eta_y^2. \tag{23}$$

Now

$$\frac{d\zeta}{dz} = \xi_x + i\eta_x$$

and

$$\overline{\frac{d\zeta}{dz}} = \xi_x - i\eta_x.$$

It follows that

$$E = e^Q = \frac{d\zeta}{dz}\overline{\frac{d\zeta}{dz}}. \tag{24}$$

In the event that the mapping function $\zeta(z)$ is not single-valued, the z-plane may be replaced by a Riemann surface so that a one-to-one correspondence in the mapping from the Riemann surface to the ζ-plane is preserved.

PART II. UNIFORM FLEXURE OF A NATURALLY CURVED SPIRAL BEAM

4. General Theory

Consider the isometric coordinate system derived from the mapping

$$\zeta = e^{\left(\frac{1-ik}{2}\right)z} \tag{25}$$

where k is a real constant and

$$\zeta = re^{i\theta}. \tag{26}$$

Separating real and imaginary parts gives

$$\begin{aligned} r &= e^{\frac{1}{2}(x+ky)} \\ \theta &= \tfrac{1}{2}(y - kx). \end{aligned} \tag{27}$$

By eliminating x and y in turn from (27) we obtain the two families of orthogonal, logarithmic spirals

$$\left.\begin{aligned} r &= e^{\left(\frac{1+k^2}{2}\right)x + k\theta} \\ r &= e^{\left(\frac{1+k^2}{2k}\right)y - \frac{\theta}{k}} \end{aligned}\right\} \tag{28}$$

which constitute the coordinate lines of the ζ-plane. From (24)

$$E = \left(\frac{1 + k^2}{4}\right)e^{x+ky}. \tag{29}$$

As a result,

$$Q = \log\left(\frac{1 + k^2}{4}\right) + x + ky. \tag{30}$$

It can be shown by introducing a linear mapping that a spiral with a given k value may be mapped into any other spiral with the same k value by magnification alone. Thus, the two lateral edges of a beam are geometrically similar. However, in general, for different values of k, the corresponding beams are not geometrically similar. Accordingly, k is adopted as a shape parameter. Furthermore, a given spiral may also be mapped into another spiral with the same k value by rotation about the origin alone. The value of the x-coordinate in the first of Eqs. 28 merely fixes the spiral in the plane at a given orientation. Consequently, without loss of generality we may choose as a reference boundary for all beams of a given k the boundary $x = 0$. Any other spiral of the same k can be specified by the angle β through which it must be rotated counterclockwise to coincide with the reference spiral. If the value of the x-coordinate specifying the outer boundary of the

spiral is $x = \delta$, the value of β may be shown to be

$$\beta = \frac{1 + k^2}{2k}\,\delta. \tag{31}$$

A typical spiral beam is shown in Fig. 1.

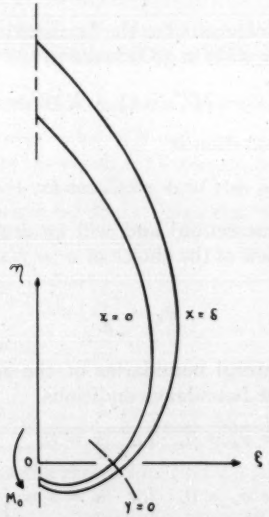

FIG. 1. Part of a typical spiral beam.

5. The Stress Function and Boundary Conditions

Making use of (30) and (16) the stresses in spiral coordinates are

$$\left.\begin{aligned}
\sigma_x &= \frac{4}{1+k^2}\,e^{-(x+ky)}\left(\varphi_{yy} - \frac{k}{2}\,\varphi_y + \tfrac{1}{2}\varphi_x\right)\\
\sigma_y &= \frac{4}{1+k^2}\,e^{-(x+ky)}\left(\varphi_{xx} + \frac{k}{2}\,\varphi_y - \tfrac{1}{2}\varphi_x\right)\\
\tau_{xy} &= \frac{4}{1+k^2}\,e^{-(x+ky)}\left(-\varphi_{xy} + \frac{k}{2}\,\varphi_x + \tfrac{1}{2}\varphi_y\right)
\end{aligned}\right\}. \tag{32}$$

Substituting these stresses in the equation of compatibility, (16), gives

$$\nabla^2\left[e^{-(x+ky)}(\varphi_{xx} + \varphi_{yy})\right] = 0 \tag{33}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

We shall seek a particular solution of the form

$$\varphi = f(x) \tag{34}$$

and show that this choice satisfies the boundary conditions for uniform flexure.[5] Substituting (34) in (33) leads to

$$f^{iv} - 2f'' + (1 + k^2)f'' = 0.$$

The solution of this equation is

$$f(x) = C_1 + C_2 x + e^x(C_3 \cos kx + C_4 \sin kx). \tag{35}$$

The constant C_1 is unessential and will be dropped from subsequent considerations. In view of the choice of $\varphi = f(x)$ we see from (32) that

$$\sigma_z = \frac{\tau_{xy}}{k}. \tag{36}$$

To insure that the lateral boundaries of the spiral beam are free of stress we introduce the boundary conditions

$$\tau_{xy} = \sigma_z = 0, \quad \text{for} \quad x = 0 \tag{37}$$

$$\tau_{xy} = \sigma_z = 0, \quad \text{for} \quad x = \delta = \frac{2k\beta}{1 + k^2} \tag{38}$$

for all values of y. By substituting (35) into either the first or third of Eqs. 32 it can be shown that the boundary conditions (37) and (38) are satisfied if the pair of equations

$$\left.\begin{aligned} C_3 + kC_4 + C_2 &= 0 \\ mC_3 + nC_4 + e^{-\delta}C_2 &= 0 \end{aligned}\right\} \tag{39}$$

is satisfied, where for convenience,

$$\left.\begin{aligned} m &= \cos k\delta - k \sin k\delta \\ n &= k \cos k\delta + \sin k\delta \end{aligned}\right\}. \tag{40}$$

The constants C_3 and C_4 may be expressed in terms of C_2, postponing

[5] By uniform or pure flexure we shall understand a distribution of stress at any cross-section of a beam that is statically equivalent to a couple.

temporarily the significance of the constant C_3. Thus,

$$C_3 = \frac{C_2(ke^{-k} - n)}{n - km} \tag{41}$$

$$C_4 = \frac{C_2(m - e^{-k})}{n - km}. \tag{42}$$

To obtain a third equation for the determination of the constant C_2 we must investigate the resultant forces on a beam cross-section.

6. Evaluation of the Resultant Forces on a Beam Cross-Section $y = Constant$

Consider an arbitrary curve AB referred to the rectangular axes (ξ, η). We shall denote the resultant forces on curve AB in the ξ and η directions by F_ξ and F_η, respectively, and the moment of these forces about the coordinate origin by M_o. Using this notation the forces and moment are known (5) to be

$$F_\xi = \left.\frac{\partial \varphi}{\partial \eta}\right|_A^B \tag{43}$$

$$F_\eta = -\left.\frac{\partial \varphi}{\partial \xi}\right|_A^B \tag{44}$$

$$M_o = \left.\varphi\right|_A^B - \left[\xi \frac{\partial \varphi}{\partial \xi} + \eta \frac{\partial \varphi}{\partial \eta}\right]_A^B. \tag{45}$$

In view of the choice of the stress function we have $\varphi_y \equiv 0$. Then,

$$\frac{\partial \varphi}{\partial \eta} = \varphi_x \frac{\partial x}{\partial \eta}, \quad \frac{\partial \varphi}{\partial \xi} = \varphi_x \frac{\partial x}{\partial \xi}. \tag{46}$$

If we now consider the resultant force on any beam cross-section $y = y_0$, the points A and B of the arbitrary curve have the coordinates $(0, y_0)$ and (δ, y_0), respectively. Therefore, making use of (46),

$$F_\xi = \varphi_x \left.\frac{\partial x}{\partial \eta}\right|_{0, y_0}^{\delta, y_0} \tag{47}$$

$$F_\eta = -\varphi_x \left.\frac{\partial x}{\partial \xi}\right|_{0, y_0}^{\delta, y_0}. \tag{48}$$

Referring to the stresses in (32) and the boundary conditions (37) and (38), it is seen that the satisfaction of the boundary conditions on the lateral edges of the beam as expressed by (39) is equivalent to the requirement that $\varphi_x = 0$ on the lateral edges of the beam. Thus, we

have the result from (47) and (48) that

$$F_\xi = F_\eta = 0. \tag{49}$$

In the same way the bending moment M_o reduces to

$$M_o = \varphi\Big|_{0,y_0}^{\delta,y_0} = \frac{C_2}{1+k^2}\left[\delta(1+k^2) + 2k(\cot k\delta - \cosh\delta \csc k\delta)\right]. \tag{50}$$

The significance of the constant C_2 is thus established in terms of the moment on the beam at the vertex of the spiral. We have

$$C_2 = \frac{M_o(1+k^2)}{\delta(1+k^2) + 2k(\cot k\delta - \cosh\delta \csc k\delta)}. \tag{51}$$

Let

$$N = \delta(1+k^2) + 2k(\cot k\delta - \cosh\delta \csc k\delta). \tag{52}$$

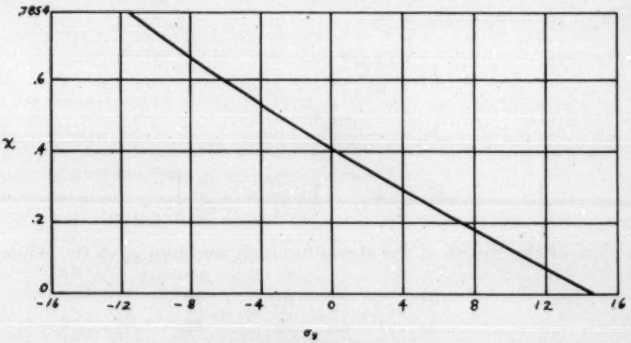

FIG. 2. Bending stress distribution at section $y = 0$.
$k = 1$, $\beta = 0.7854$, $M_o = 1$.

The stresses can be written in terms of the bending moment as follows:

$$\sigma_x = \frac{2M_o e^{-ky}}{N}\left[e^{-z} - \cos kx + \left(\frac{\cos k\delta - e^{-\delta}}{\sin k\delta}\right)\sin kx\right] \tag{53}$$

$$\tau_{xy} = k\sigma_x \tag{54}$$

$$\sigma_y = \frac{4M_o e^{-ky}}{N\sin k\delta}\left[\begin{array}{l}(k\cos k\delta - \tfrac{1}{2}\sin k\delta - ke^{-\delta})\cos kx \\ \quad + (\tfrac{1}{2}\cos k\delta + k\sin k\delta - \tfrac{1}{2}e^{-\delta})\sin kx \\ \quad - \tfrac{1}{2}e^{-z}\sin k\delta\end{array}\right]. \tag{55}$$

In Figs. 2 and 3 the bending and shearing stress distributions at the

cross-section $y = 0$ are shown for a spiral beam defined by the shape parameters $k = 1$, $\beta = \pi/4$. It is to be noted that the origin at which point the concentrated moment is applied is a singular point of infinite stress. In view of St. Venant's principle the stresses at a point sufficiently far removed from the origin will be unaffected by the manner of application of the moment and will be given by the above equations.

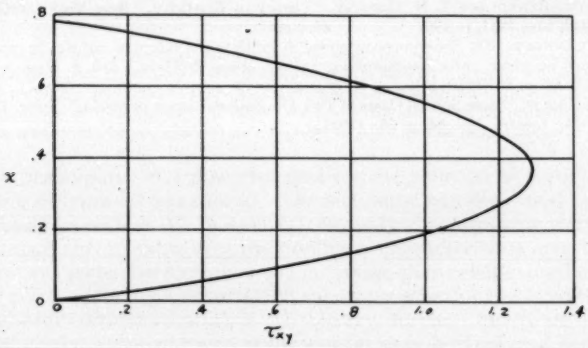

FIG. 3. Shearing stress distribution at section $y = 0$. $k = 1$, $\beta = 0.7854$, $M_o = 1$.

7. *Limiting Cases of the Solution for a Spiral Beam*

By allowing the shape parameter k to approach zero, the spiral coordinates become isometric polar coordinates in the limit. It may be shown that the stresses given by (53), (54), and (55) are those resulting from the uniform flexure of a sector of a circular ring, the solution of which is well known by other methods. Likewise, by letting k approach infinity, in the limit the spiral coordinates become isometric polar coordinates, the role of the x and y coordinate lines, however, is interchanged. A spiral beam in the limit becomes a wedge. The stresses obtained in this case are those resulting from a moment applied to the wedge at its vertex. The solution of this problem by other methods has been given by Fillunger (6) and Inglis (7).

Acknowledgment

The author expresses his appreciation to Dr. H. L. Langhaar, under whose direction this work was conducted.

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ANALYSIS OF NONLINEAR SERVOS BY PHASE-PLANE-DELTA METHOD

BY

ROBERT N. BULAND [1]

SYNOPSIS

The growing interest in the synthesis of nonlinear servomechanisms demands better analytical techniques. The phase-plane-delta method is a graphical, step-by-step technique developed to solve nonlinear, non-homogeneous differential equations of second order. Higher order nonlinear equations can also be solved providing that the equation can be re-expressed as a number of equations of second order or less.

The phase-plane is a powerful tool for the solution of second order nonlinear differential equations. The technique has been used by a few servo analysts such as McDonald (1)[2] to analyze nonlinear systems and, more important, to show that the nonlinear characteristics may be used to improve servo performance. The phase-plane-delta method is a special phase plane technique which shows promise for the investigation of nonlinear servomechanisms. Arbitrary forcing functions can be handled easily, and equations of high order may be analyzed providing the equation is factorable.

The phase-plane-delta method or δ-method was formulated by Jacobson (2) and is a generalization of the Lienard method (3, 4). E. Q. Smith of United States Naval Air Missile Test Center, Point Mugu, California has made some modifications for application to non-homogeneous vibrations problems.

The method, if accomplished manually, is a step-by-step graphical process. Accuracy is a function of the number of steps. Since nonlinear coefficients and forcing functions are introduced graphically, solutions are quite rapid.

DEVELOPMENT OF METHOD

The δ-method is based upon the assumption that a second order differential equation can be so expressed that the slope of the phase-plane curve representing this equation can be predicted at any time after the beginning of the problem.

For clarity a linear second order equation will be used to demonstrate the method. It will then be shown that the addition of nonlinearities will present no problem providing they can be represented graphically.

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[1] United States Naval Air Missile Test Center, Point Mugu, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Consider the equation

$$M\ddot{x} + C\dot{x} + Kx = F(t) \tag{1}$$

which can be written

$$\ddot{x} + \frac{K}{M}x + \frac{CK}{MK}\dot{x} - \frac{K}{MK}F(t) = 0$$

if

$$\omega_n^2 = \frac{K}{M};$$

then

$$\ddot{x} + \omega_n^2 x + \omega_n^2\left[\frac{C}{K}\dot{x} - \frac{1}{K}F(t)\right] = 0$$

$$\delta = \left[\frac{C}{K}\dot{x} - \frac{1}{K}F(t)\right]$$

$$\ddot{x} + \omega_n^2[x + \delta] = 0. \tag{2}$$

All equations which are solved by the δ-method must be reduced to the form of Eq. 2.

The phase-plane coordinates are defined as

$$x = x \tag{3a}$$

$$v = \dot{x}/\omega_n. \tag{3b}$$

From Eq. 3*b*

$$\dot{x} = \omega_n v$$

$$\ddot{x} = \omega_n\, dv/dt = \omega_n \frac{dx}{dt}\frac{dv}{dx} = \omega_n^2 v \frac{dv}{dx}. \tag{4}$$

Substituting into Eq. 2

$$\omega_n^2 v \frac{dv}{dx} = -\omega_n^2[x + \delta]$$

$$\frac{dv}{dx} = -\frac{[x + \delta]}{v}. \tag{5}$$

Equation 5 defines a slope in the phase plane at a point P' with the coordinates v, $[x + \delta]$. Figure 1(*a*) indicates the geometry of the problem. δ can be defined as a distance measured along the x axis or as a vector of length δ and angle 90° if the phase axis be oriented so that x is vertical and v is horizontal. Note that this orientation is not conventional.

The polar coordinates of point P' of Fig. 1(*a*) are $\rho/\underline{\theta}$ where

$$\rho = \sqrt{v^2 + [x + \delta]^2}.$$

Then by vector addition

$$\vec{\gamma} = \vec{\rho} - \vec{\delta}$$

where $\gamma/\underline{\mu}$ are the polar coordinates for point P of Fig. 1.

Equation 5 defines the direction that point P' will move if ρ is advanced through an angle, $d\theta$, generating an arc, $ds = \rho d\theta$. If δ is held constant while this arc is generated then ds is also generated from point P. Point P lies on the phase-plane curve representing the equation under study. Thus the arc is a close approximation to the desired solution. Since time solutions are desired let us find a relation between

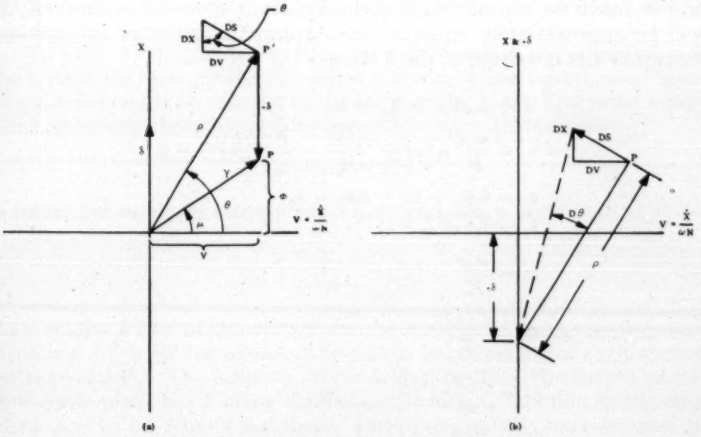

FIG. 1. Geometry for phase-plane-delta method.

θ and time. It can be seen from Fig. 1(a) that the triangle formed by ds, dx, and dv is similar to the triangle formed by ρ, v, and $x + \delta$. Therefore:

$$\frac{ds}{\rho} = \frac{dx}{v}.$$

It is also known that:

$$\rho d\theta = ds.$$

Then by substitution

$$d\theta = \frac{dx}{v} = dx \cdot \omega_n \frac{dt}{dx}$$

$$d\theta = \omega_n dt. \tag{6}$$

Thus there exists a simple relationship between angular displacements in the phase plane and time. Also, all of the difficult elements in the equation are grouped together in one term, δ, which affects the phase plane in a straightforward manner.

As an example of a nonlinear equation consider

$$M\ddot{x} + f_1(x)\dot{x} + f_2(x) = F(y)F(t) \tag{7}$$

where the forcing function varies both as a function of the input amplitude and time. To make a δ solution a Kx must be added to each side of the equation. Any value of K may be chosen without affecting the results of the solution, however experience indicates that the solution will be much more rapid if the spring constant is chosen so that $\sqrt{K/M}$ will be approximately equal to the natural frequency of the system. Equation 7 is rewritten in the δ form

$$\ddot{x} + \omega_n^2[x + \delta] = 0$$

$$\delta = \frac{\omega_n v}{K} f_1(x) + \frac{f_2(x)}{K} - \frac{F(y)F(t)}{K} - x$$

$$\delta = \delta_1\delta_2 + \delta_3 - \delta_4\delta_5 - \delta_6$$

$$\delta_1 = f_1(x)$$

$$\delta_2 = \frac{\omega_n v}{K}$$

$$\delta_3 = \frac{f_2(x)}{K}$$

$$\delta_4 = \frac{F(y)}{K}$$

$$\delta_5 = F(t)$$

$$\delta_6 = x.$$

Thus the complexities of Eq. 7 have been reduced to graphical plots and a tabulation to combine them as the initial conditions of the problem and the phase-plane plot dictate.

When the plotting is to be done by hand with protractor and compass, the geometry may be handled in a slightly different manner. A positive δ is laid off on the negative x axis. Then P will lie as indicated in Fig. 1(*b*). The solution takes a step-by-step form. A value of δ is computed for average conditions during the step. Then the step, $d\theta$, is generated, $d\theta$ being chosen to represent a convenient unit of time small enough to achieve the desired accuracy. At the end of the step, δ is readjusted for the next step, and the process is repeated.

EXTENSION OF METHOD

The limitation of phase-plane techniques to second order equations has been emphasized in the literature. While many servomechanism systems are of higher order than second, it is significant that such systems are usually represented by a series of transfer functions of low order. Generally all transfer functions are presented in factored form. In other words, equations for complicated systems are usually expressed in a manner which allows interpretation as a series of first and second order equations. If the phase-plane plot for the δ-method is considered as a transfer block the input may be any arbitrary forcing function and the output can be either position or velocity or both. A series of δ-method transfer blocks could represent a high order system and a solution could be made considering many nonlinearities which would normally be neglected.

To solve a complicated system by the δ-method the equations for the system are re-expressed as a series of second order equations. Since all equations must be reduced to the form of Eq. 2, any first order equations remaining must be raised to second order. For example

$$\dot{x} + ax = Ky$$

is raised one order by introducing a new variable, z, such that

$$\dot{z} = x$$
$$\ddot{z} + a\dot{z} = Ky.$$

Any nonlinearities of the system can be considered in the appropriate equation. A separate phase-plane plot is constructed for each second order equation. The solution proceeds step-by-step, the output of the first phase-plane block being applied as the forcing function to the next block and so on around the loop. Iterative methods may be used to improve accuracy. Solutions are relatively rapid and can be carried out by subprofessional personnel. Solutions of this type should prove especially useful as checks for analog computer problems.

Although, normally, the δ-method is best suited to manual computations, certain conditions might arise where it would be advantageous to make automatic solutions by means of an analog computer circuit. A two integrator oscillator with an additional input, δ, will generate the phase-plane plot. One such application might be the representation of an aircraft response where the dynamic characteristic can be represented as a second order equation but the gain is a function of lift and velocity.

Example I

The first problem involves an on-off rate sensitive controller which

produces either no torque or full torque output. Refer to Fig. 3 for a block diagram representation of the servo system.

If ϕ = angular rate input to controller
γ = angular output
K_1 = threshold sensitivity of controller

then

$$\gamma = 0 \qquad \phi < K_1$$
$$\gamma = 20t \qquad \phi \geq K_1.$$

Since the region of interest in the transient response from some initial condition of ϕ to $\phi = K_1$, the controller can be considered as a ramp function generator and the characteristic of the second block can be analyzed by linear theory. The equation for the system can be written

$$\ddot{\phi} + 7.4\dot{\phi} = -116.3\gamma \tag{8}$$
$$\ddot{\phi} + 7.4\dot{\phi} = -116.3(20t)$$
$$s\dot{\phi} - \dot{\phi}(0) + 7.4\dot{\phi} = -\frac{116.3 \cdot 20}{s^2}$$
$$\dot{\phi} = \frac{\dot{\phi}(0)}{[s + 7.4]} - \frac{116.3 \cdot 20}{s^2[s + 7.4]}.$$

Thus, for the region of interest a linear solution exists and a check on the accuracy of the δ-method can be made.

For the δ solution, Eq. 8 is modified by adding a displacement term. Note that the coordinates of the phase-plane plot of Fig. 3 are $\dot{\phi}$ and $\alpha = \dot{\phi}/\omega_n$. The mass and damping constant are known for this system. Also it is known that the natural frequency of the system is about 12.5 radians per second. Therefore $\omega_n^2 = 156$ radians per second is used, since it has been found that the solution is quicker and the plot cleaner if the natural frequency of the system is used where a choice exists. Therefore the equation becomes

$$\ddot{\phi} + 7.4\dot{\phi} + 156\phi - 156\phi + 116.3\gamma = 0$$
$$\ddot{\phi} + 156[\phi + \delta] = 0$$
$$\phi \equiv 12.5\alpha$$
$$\delta = \delta_1 - \delta_2 + \delta_3$$
$$\delta_1 = \frac{7.4}{12.5}\alpha = 0.594\alpha$$
$$\delta_2 = \phi$$
$$\delta_3 = \frac{116.3}{156} = 0.745\gamma.$$

The time increment is chosen to equal 0.02 seconds.

$$d\theta = \omega_n dt$$

$$d\theta = \frac{0.02 \cdot 12.5 \cdot 180}{\pi} = 14°.$$

The curves of δ_1, δ_2 and δ_3 have been plotted in Fig. 2. The phase-

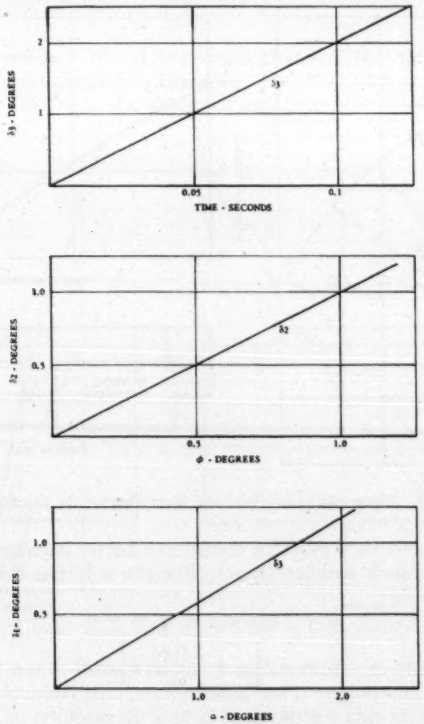

FIG. 2. Curves of δ_1, δ_2 and δ_3 for Example I.

plane plot is shown in Fig. 3. The construction lines have been dotted in. The time plot indicates that the accuracy of the δ-method is good.

Example II

The next example problem is a servo system which approximates a second order equation with nonlinear damping. It is desired to find

the transient response due to an initial condition. The first solution method is similar to the one used by Lewis (5) in which the damping is considered to vary in steps or zones and the linear equation is solved during the zone. At the end of the zone a new value of damping is used and the initial conditions are taken from the last point of the preceding zone. As Lewis points out, such a solution is extremely laborious. With the δ-method, the same solution was made in two

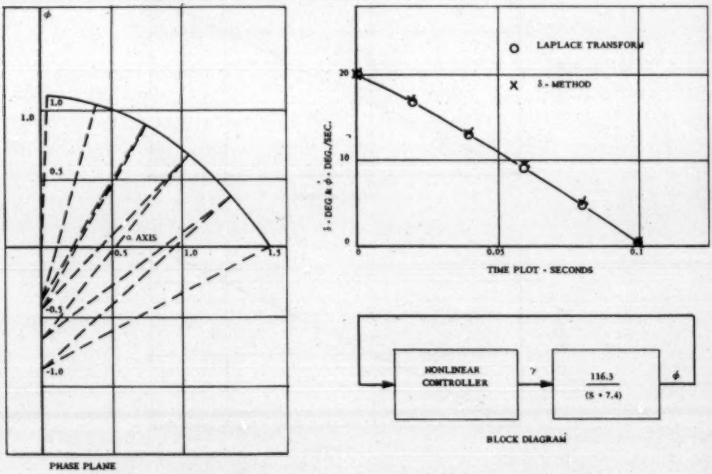

FIG. 3. Phase-plane, time plot and block diagram for Example I.

hours and the accuracy must be considered better because the damping was varied in much smaller steps. For the solution the equation is:

$$\ddot{\phi} + f(\phi)2\omega_n\phi + \omega_n^2\phi = 0$$

$$\ddot{\phi} + \omega_n^2\left[\phi + \frac{2f(\phi)}{\omega_n}\dot{\phi}\right] = 0$$

$$\alpha \equiv \dot{\phi}/\omega_n$$

$$\delta = \delta_1\delta_2$$

$$\delta_1 = 2\alpha$$

$$\delta_2 = f(\phi)$$

$$dt = d\theta/\omega_n$$

$$dt = 0.0014\, d\theta.$$

For convenience δ_1 and δ_2 are plotted in the phase plane of Fig. 4. Note that the plot has an abrupt change of direction near $\phi = 20$ deg. Within such a region the size of the steps should be reduced because Jacobsen points out that in regions of rapid change the δ-method is inherently inaccurate.

The results of the two methods of solution are compared on the time plot of Fig. 4.

Example III

The last problem is the most complicated and the only check is an approximate linear solution. However, the method has already been well verified so that only the choice of parameters is questionable.

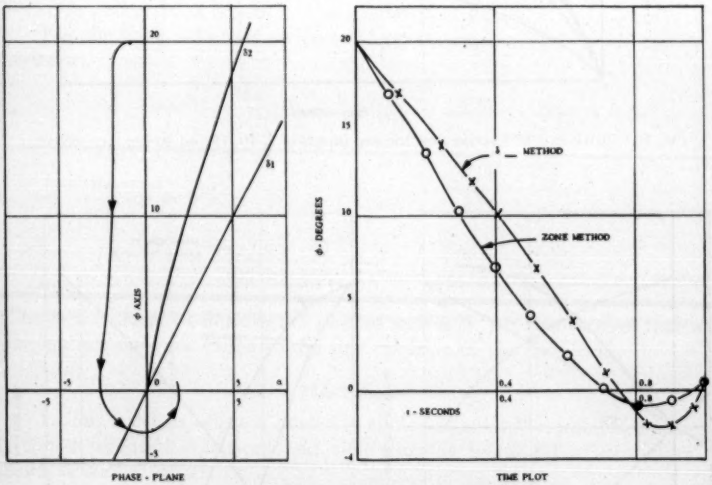

FIG. 4. Phase plane and time plot for Example II.

The device under consideration is quite similar to the controller of Example I. However this device has a linear region and a low saturation point. The response to a step function for this controller can be approximated by an exponential rise with a time constant of 0.3 sec. The saturation curve was found experimentally and the exponential time constant was found by averaging several transient curves. The resulting family of transient curves is plotted in Fig. 5. The block diagram is the same as in Fig. 3. Writing the equation for the second block

$$\ddot{\phi} + 7.4\phi = f(\phi)[1 - e^{-3.33t}].$$

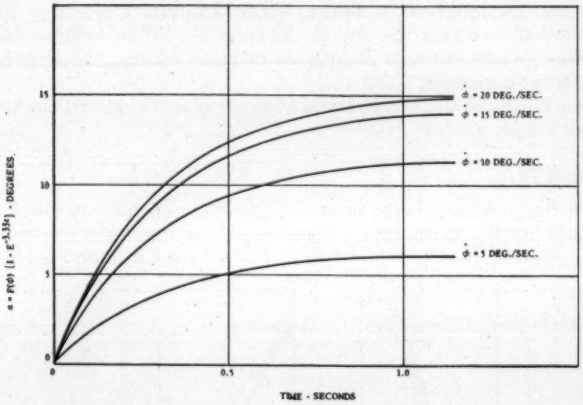

FIG. 5. $f(\dot{\phi})(1 - e^{-3.33t})$ *versus* time for step inputs of 5, 10, 15 and 20 deg. per second.

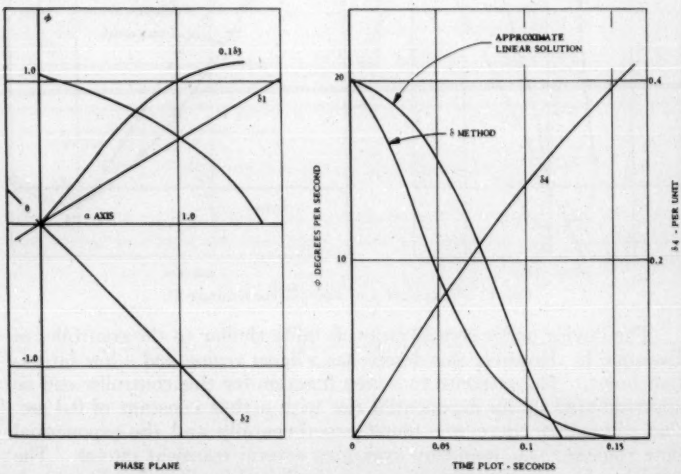

FIG. 6. Phase plane and time plot for Example III.

Again the natural frequency will be assumed to be 12.5 radians per second. The equation is of the same form as the first example and can be written

$$\ddot{\phi} + 156[\phi + \delta] = 0$$
$$\delta = \delta_1 + \delta_2 - \delta_3\delta_4$$
$$\delta_1 = 0.594\alpha$$
$$\delta_2 = -\phi$$
$$\delta_3 = f(\dot{\phi})$$
$$\delta_4 = [1 - e^{-3.33t}].$$

A time increment of 0.02 sec. was again chosen so that $d\theta = 14°$. The solution proceeds as before and is indicated in Fig. 6.

For the linear solution the closed loop response of the system can be written:

$$s^2\phi - s\phi(0) - \frac{d\phi}{dt}(0) + 2.65(s\phi - \phi(0)) + 156\phi = 0$$
$$\phi(0) = 20 \text{ deg./sec.}$$
$$\frac{d\phi}{dt}(0) = 0$$
$$\phi(s) = \frac{20 \cdot 2.65}{s^2 + 2.65s + 156} + \frac{20s}{s^2 + 2.65s + 156}.$$

The two transient solutions are plotted in Fig. 5. It can be seen that a fair correspondence exists to the first crossing of the time axis.

CONCLUSIONS

1. The method offers a powerful means of solving nonlinear second order differential equations and time varying linear differential equations of second order.

2. Most servo systems can be expressed as a series of second order expressions. The method can then be used to solve for the transient response since any arbitrary forcing function is acceptable for solution by this method. Therefore the output of one phase-plane plot can be used as the input to the next phase-plane plot until the loop is closed.

3. Nonlinearities and forcing functions are introduced graphically, in many cases, directly from experimental data. Thus alleviating the necessity for finding an analytical expression for them.

4. The method may be mechanized to solve equations automatically on an analog computer. This might prove advantageous in solving complicated nonlinear problems.

Acknowledgment

The author wishes to express his gratitude to E. Q. Smith of the United States Naval Air Missile Test Center, Point Mugu, California, for his support and encouragement in learning and using this method.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SEMICONDUCTING INTERMETALLIC COMPOUNDS

Investigations by the National Bureau of Standards into the fundamental properties of semiconductors have revealed that certain intermetallic compounds show promise of extended use in solid state electronic devices. Current research, under the direction of R. G. Breckenridge of the Bureau's solid state physics laboratory is concerned principally with the conductivity and the Hall effect of such metal compounds as indium antimony (InSb) and aluminum antimony (AlSb). Of immediate importance is the fact that these combinations may have equal or greater utility than the germanium and silicon semiconductors presently in large demand. More extensive investigations are expected to disclose many useful and interesting characteristics of these hitherto relatively unexplored materials.

The successful development during the war of silicon and germanium diodes and the subsequent discovery of the germanium triode marked the beginnings of the phenomenal expansion of solid state electronics. The products of this growth in semiconductor technology have supplemented vacuum tubes in many applications and even replaced the tubes in some equipment. Transistors and semiconducting rectifiers have contributed immeasurably to the development of more rugged and compact radio, radar, and other electronic devices. However, the realization of the full industrial potential of these units has been seriously handicapped by material limitations and difficulties in the preparation of the single crystals of germanium and silicon needed for newer semiconductor devices. Thus, NBS research has been directed toward an investigation of intermetallic compounds that exhibit the same or better semiconductor characteristics and that are more easily manufactured than the metals currently in use.

Early NBS research on semiconducting materials involved the preparation of "grey tin," a form of tin stable below 13.2° C. It was reasoned that this material should be a semiconductor because its diamond structure resembles that of silicon and germanium. Efforts were made to condense tin vapor on a substrate material having a crystal lattice that properly matched grey tin, which would thus assist in the growth of the low-temperature form. The substrate chosen was indium antimony, a compound which, under X-ray examination, has a crystal structure unlike either indium or antimony but matches grey tin almost exactly. Although the investigation was unsuccessful (in regard to the

* Communicated by the Director.

production of grey tin), the physical properties of the InSb suggested that it might also have useful semiconducting characteristics. Later investigations on high-purity samples abundantly confirmed the hypothesis that indium antimony was actually a semiconductor and, in addition, was found to have very high carrier mobilities (speed with which the carrier of electron current moves in electric fields).[1]

The current research program at NBS has involved an investigation of the Hall effect and the conductivity of several such semiconducting materials. Related phenomena, such as optical absorption spectra, photoconductivity, and rectification effects, are also under observation. All of the studies thus far have been performed with polycrystalline samples; but when methods of growing single crystals, presently being tried by NBS, have been successful, each experiment will be repeated on these crystals.

The importance of the investigations of the Hall effect and conductivity is that the results give an indication of the number of charge carriers present and the charge carrier mobility. The Hall effect occurs when a magnetic field is applied perpendicular to an electric current flowing in a conducting material. A voltage is produced within the conductor whose vector direction is mutually perpendicular to the current and magnetic field. The magnitude of the voltage depends upon the number of charge carriers in the conductor; its sign is determined by the type of conduction—either by "holes" or by free electrons.

Most semiconductors depend for their conductivity on impurities within the crystal lattice; that is, they are extrinsic semiconductors. For one type of impurity, the foreign atoms are ionized thermally and thus provide conduction electrons. Impurities of this sort are called "donors" and the material is an "n-type" semiconductor, which indicates that the sign of the charge carriers is negative. Another type is one in which the foreign atom is ionized by accepting an electron ejected from the normal energy level of the material, thus leaving a hole in that level. This vacant electron site will also move in an electric field and behaves like an electron with an effective positive charge. The situation is analogous to a filled checkerboard on which no moves are possible until one checker is removed from the board. Conduction is then by "holes" and the name "p-type" indicates the positive sign of the charge carriers. Impurities of this sort are known as "acceptors." The temperature region in which the foreign atoms act as the source of electrical conduction is called the "impurity" range. At sufficiently elevated temperatures electrons may be thermally excited in the pure material itself. This temperature region is called the "intrinsic" range.

High charge carrier mobility is a fundamental property required for

[1] The first public announcement of these results was made at the meeting of the Solid State Advisory panel composed of representatives from leading research laboratories, November 21, 1952.

transistor action. In a point contact transistor the emitter injects "holes" into the semiconductor. These holes migrate under the influence of the applied electric field to the collector where their presence influences the flow of electrons from the collector through to the base electrode. The ability of the transistor to follow high frequencies depends on how fast the holes can travel from the emitter to collector; the shorter the time required, the higher the frequency. In a typical sample of high grade germanium, the mobility is about 3000 cm²/volt sec. For ordinary-size electrodes and spacings this restricts the highest frequency to about 10 or 20 Mc. By making the contact points very small and by putting them very close together, the upper-frequency limit has been raised to about 100 Mc, but with a sacrifice of power handling ability.

Another fundamental property governing the migration of the injected holes is the purity of the semiconductors. In a highly impure sample the injected charge carrier will become trapped at a foreign atom and disappear before it has had time to migrate to the vicinity of the collector. Thus, the charge carrier will not contribute to transistor action. The number of charge carriers found in the Hall effect and conductivity measurements is a direct indication of the number of such trapping centers. At room temperatures, carrier mobility is primarily a function of the nature of the crystal lattice, with a small decrease in mobility from impurity scattering of the electron. The number of charge carriers is primarily a function of the sample purity. Consequently, if a material is found to have a high lattice mobility, its properties can be improved by purification, but, on the other hand, a material with a low lattice mobility cannot be markedly improved.

The samples of indium antimony investigated by the National Bureau of Standards have not been sufficiently pure to give transistor action, but their mobility (20,000 cm²/volt sec) is about seven times that of germanium. With sufficiently pure samples, transistor action should be possible at much higher frequencies and at higher power levels. The mobilities in the other compounds studied (GaSb, AlSb, CdSb), while high, are apparently not as high as in InSb and may not function as well at high frequencies.

Another parameter, important with respect to the performance of practical diode and triode semiconductors, is the change in properties with temperature. In a semiconductor the number of charge carriers increases exponentially with temperature. In the impurity range this change is slow because the activation energy ΔE appearing in the exponential relationship for semiconductor conductivity, $\sigma = \sigma_0 e^{-\Delta E/2kT}$, is small. At higher temperatures the intrinsic range is reached. Here changes with temperature are much more rapid because the energy is larger. For a material to be useful, the intrinsic range should not be reached at temperatures normally encountered, that is, below 200° C. The activation energy of germanium is 0.75 e.v., its intrinsic range thus

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starts at about 60° C. for samples of normal purity. Silicon samples of the usual purity, with an activation energy of about 1.1 e.v., do not enter the intrinsic range until a temperature of approximately 350° C. is attained. Unfortunately, pure silicon in single crystal form is very hard to obtain because of its chemical activity and also its very high melting point. Although rectifiers have been made in the laboratory from single crystals of silicon, none has been commercially produced.

From investigations of the temperature variation of Hall effect and conductivity, it was found that the activation energy of indium antimony is about 0.40 e.v. This value is too low to allow transistor action at elevated temperatures. Gallium antimony, on the other hand, has an activation energy of about 0.86 e.v.—somewhat greater than germanium; and aluminum antimony appears to have an activation energy greater than 1 e.v.—slightly larger than the energy of silicon. It is assumed that from the variety of intermetallic compounds available it will be possible to find suitable combinations with proper energy to yield the desired semiconductor characteristics.

NBS research on single crystals is aimed at the preparation of junction-type diodes and triodes, particularly the p-n junction diode—conduction first by holes and then electrons. The performance of these devices, especially in regard to their power handling ability, is markedly superior to point contact units. In addition, the theory and practice, especially for diodes, is well understood. It is to be hoped that the improved ease of handling GaSb over Si will permit the preparation of a good high-temperature diode and eventually n-p-n transistors, although this last stage may be difficult.

From a purely practical standpoint, the most important feature of the NBS research program is the investigation of materials with equal or higher charge carrier mobilities than germanium. From a longer-range viewpoint it should be realized that practical solid state electronic devices no longer need be restricted to Si and Ge; the existence of a wide variety of compounds whose properties are virtually unexplored, but which give every indication of being useful, forecasts a latitude in design that was not before possible.

NOTE: For additional information on semiconductors and solid state electronics see "Solid State Electronics," *NBS Tech. News Bull.*, Vol. 35, No. 2, p. 22 (1951).

LABORATORY MEASUREMENT OF CORROSION IN SOILS

A recent study by the National Bureau of Standards has shown that reliable data on the corrosion of ferrous metals in soils of various types can be obtained in the laboratory in a period of six months. Usually field tests lasting 10 years or more are required. In the NBS investigation, which was conducted by W. J. Schwerdtfeger of the Bureau's corrosion laboratory, corrosion was produced in small laboratory cells

by differential aeration of metal disks in contact with soil within the cells. The results for 15 different soils showed good correlation with the corrosion observed on wrought specimens in long-term exposure tests. The technique used by the Bureau should thus provide a valuable basis for predicting the service of iron and steel structures exposed to various soils.

The corrosion of metals in many soils is attributed chiefly to differences in the accessibility of oxygen to the surface. Differences in oxygen concentration create differences in the potential of various areas, resulting in the flow of electric currents through the soil from the oxygen-deficient areas (anodes) to those areas (cathodes) receiving more oxygen. The laboratory corrosion cells used by NBS made use of this principle. Each cell consisted of a cylindrical Lucite enclosure containing two short-circuited disk-shaped electrodes of the same material, iron or steel. One electrode was solid, the other perforated, and they were separated by a layer of moist soil. The edges and one face of each of the two electrodes were covered with bituminous paint. The layer of moist soil in contact with the coated side of the perforated electrode regulated the diffusion of air to the electrodes. The perforated electrode received a greater supply of oxygen than the solid one adjacent to which the soil was puddled, and therefore assumed a more positive potential.

If the corrosion of ferrous metals in all soils proceeded at a rate that was constant with time, weight-loss measurements made after a short arbitrary period of exposure should indicate relative rates of corrosion, assuming negligible cleaning errors. Actually, however, corrosion in soils may proceed at a rate proportional to time, or may sometimes cease entirely, depending on the type of soil. It was thus necessary to operate the laboratory corrosion cells for a sufficiently long period to produce effects comparable to those shown by field exposure tests.

In the NBS investigations, the corrosion cells were kept on short circuit for six months. During this time periodic measurements of cell current permitted study of the progress of corrosion with time. At the end of the period the combined weight losses of the two electrodes were taken as the measure of corrosion.

To evaluate the laboratory corrosion cell as a means for predicting corrosion in the field, samples of soil collected at 15 NBS exposure sites were used. The results were correlated with weight-loss measurements made on short lengths of steel and wrought iron pipe which had been exposed for 10 years at the same 15 localities. An empirical equation relating the laboratory and field data indicated that the laboratory measurements could be used to predict field weight losses with root-mean-square error of about ± 4 oz/ft^2 for corrosion rates ranging from 2.5 to 30 oz/ft^2 for the 10-year exposure period. Because the 15 soils covered the range of corrosivity of the soils of the United States, this empirical relationship may be applicable to any soil.

Because some underground structures, such as piping systems and storage tanks, usually fail as a result of pitting, this factor was also considered. Analysis of the field data showed a good correlation between weight losses and maximum pit depths on wrought ferrous specimens which had been exposed for 10 years at 58 NBS exposure sites. These data were then combined with the first empirical equation to obtain a second equation which can be used to predict, with an average error of ± 18 mils, the depth of the deepest pits occurring on wrought iron or steel buried for 10 years. For the prediction of weight losses and maximum pit depths on areas greater than the area of the field specimens and for periods of exposure other than 10 years, use was made of general equations relating weight losses and maximum pit depths with length of exposure and exposed area. Average values for the constants to be used in these equations were obtained.

It was found that the corrosion cell could detect significant differences between the corrosion rates of cast iron and steel, which are generally considered to corrode at about the same rate in soils. Thus, the cell might be used to study the effects of variables, including composition upon the corrosion rates of steels. It is also reasonable to expect that laboratory corrosion cells could be used to predict the corrosion of nonferrous metals—copper, brasses, lead, zinc and possibly aluminum—in various kinds of soils. Although measurements with the laboratory cell should not be considered as replacing field exposure tests entirely, it is unlikely that materials which corrode at the same rate in the laboratory would corrode at significantly different rates in the field. Hence, the corrosion cell should serve a useful purpose in the selection of materials for field exposure tests.

NOTE: For further details, see "Laboratory Measurement of the Corrosion of Ferrous Metals in Soils," by W. J. Schwerdtfeger, *J. Research NBS*, Vol. 50, p. 329 (1953).

THE FRANKLIN INSTITUTE

MINUTES OF STATED MEETING

December 16, 1953

The Stated Monthly Meeting of The Franklin Institute was held on December 16, 1953, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:20 P.M. Approximately 210 persons attended.

The President stated that the Minutes of the November Stated Meeting were being printed in the December JOURNAL and would be submitted for approval at the next Stated Meeting.

The Secretary reported that the following nominations for election to the Board of Managers had been made in conformity with Article IV, Section 5, of the By-Laws:

Edward G. Budd, Jr.	Richard T. Nalle
W. Laurence LePage	Charles S. Redding
Hiram S. Lukens	John Russell, Jr.
John S. Malick	William M. Scott, Jr.

The President called for nominations from the floor. There being none, he declared the nominations closed. George S. Crampton, Joseph Hepburn, and Howard Stoertz were appointed Tellers.

The Secretary reported the following elections to membership since October 31st:

Active	37
Associate	34
Student	11

and a total membership, including Company and Library privileges, of 6838 as of November 30, 1953.

He read the names of those who completed twenty-five consecutive years of membership in the Institute during 1953 and who have received Silver Cards of Membership. (These are listed under the Membership Section of the JOURNAL.)

The Secretary announced that the annual James Mapes Dodge Lectures will be given on the afternoons of January 12th and 13th by Richard M. Sutton, Professor of Physics at Haverford College.

The President spoke about the exhibit of the Wright Brothers' aeronautical collection of some 240 items (including the original drawings of their 1903 and later bi-planes, their 1901 wind tunnel balances and aerofoils) which has been placed on public display in Franklin Hall, for the first time. He drew attention to the fact that Philadelphia can be proud of having a share of the classic Wrightiana. Because Orville Wright wanted the original plane and the Wright Brothers' correspondence to be located in Washington, he willed the former to the Smithsonian Institution and the latter to the Congressional Library. The Wright Brothers' experimental apparatus, note books and original drawings he willed to The Franklin Institute. The President advised that guests who had not already seen the exhibit could see it after the meeting had adjourned.

The President then introduced the speaker of the evening—Mr. Trevor Gardner, Special Assistant to the Secretary of the Air Force for Research and Development—who spoke most interestingly on "The Impact of the H-Bomb on Research and Development Planning."

The meeting adjourned at 9:20 P.M. with a rising vote of thanks to Mr. Gardner.

HENRY B. ALLEN
Secretary

MEMBERSHIP

NEW GOLD CARD MEMBER HONORED

At the Stated Meeting on November 18, The Franklin Institute honored Charles H. Howson, of Philadelphia, who has been a member continuously for fifty years. Morton Gibbons-Neff, Chairman of the Membership Committee, outlined Mr. Howson's many achievements before presenting him to the President, S. Wyman Rolph. At the conclusion of Mr. Neff's introduction, those present rose to their feet and gave Mr. Howson a standing ovation. Mr. Rolph presented a gold key and a gold membership card to Mr. Howson as tokens of his fifty years of continuous membership. Other past gold key holders present at the meeting were G. H. Clamer, 1891, and James M. Caird, 1901.

Charles H. Howson, new Gold Card Member, signs the guest register at The Franklin Institute, under the supervision of James M. Caird (left) of Troy, N. Y. and G. H. Clamer of Philadelphia, past Gold Card Members.

The following brief biography of the new Gold Card Member will serve to bring some of his accomplishments to the attention of those members who were not able to attend the meeting.

CHARLES HENRY HOWSON

A native of Philadelphia, the son of Charles and Medora Ware Howson, Charles Henry Howson was graduated from Haverford College in 1897 with a Bachelor of Arts degree and the University of Pennsylvania in 1900 with a Bachelor of Laws degree.

After being admitted to the Pennsylvania Bar in 1900, he was granted license to practice law in the various Federal courts, including the Supreme Court of the United States. Since his admission to the Bar, he has engaged continuously in the practice of law, specializing in

patent, trade-mark and copyright practice in the United States Patent Office and Federal courts.

Since 1901 Mr. Howson has been a member of the firm of Howson & Howson of Philadelphia and New York, continuing the patent practice established by his grandfather in Philadelphia in 1853.

A member of the Philadelphia Bar Association, the Pennsylvania Bar Association and of the American Bar Association, he has served as Chairman of the Patent, Trade-mark and Copyright Section of the American Bar Association.

Mr. Howson also holds membership in the American Patent Law Association and the Philadelphia Patent Law Association, of which he was a charter member and its first president. For a brief time he served as a member of the Board of Managers and Solicitor of The Franklin Institute.

Married in 1905 to May Day Yeatts of St. Davids, Pa., Mr. Howson is the father of eight children. He lives in Wayne, Pa.

The following are Gold Card Members, having belonged continuously for fifty years or more. The dates are years of election to membership.

Alexander Sellers '84
William Henry Bower '85
Fred H. Colvin '88
G. H. Clamer '91
Pierre S. duPont '92
Walter T. Lee '92
J. H. Granberry '95
Edward Woolman '95
Alan Wood, III '96
T. C. McBride '97
D. S. Jacobus '99
John J. McVey '99
E. A. Muller '99
C. C. Tutwiler '99
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Theobald F. Clark
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Franklin P. McConnell
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1914
Russell L. Brinton
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1915
Raymond Foss Bacon
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Thomas W. Elkinton
Richard Howson
S. Leonard Kent, Jr.
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W. Wallace McKaig
Walter F. Rittman

1916
Henry B. Allen
James G. Detwiler
Zay Jeffries
Thorsten Y. Olsen
Frank H. Sauer
B. E. Shackelford
H. B. Spencer
Arthur Synnestvedt
Leo Wallerstein
Charles N. Weyl

1917
A. D. Chambers
Francis J. Chesterman
Walter S. Crowell
P. T. Dashiell
William J. Fitzmaurice, Jr.
Rolfe E. Glover, Jr.
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Arthur W. Lowe
Frank S. MacGregor
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Francis F. Milne, Jr.
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1918
Charles H. Colvin
Charles H. Masland, 2nd
Walter O. Snelling

1919
Charles E. Brinley
Henry Colvin, 2nd
E. A. Eckhardt
George W. Furness
Frank H. Griffin
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H. Conrad Meyer
J. Howard Pew
Edmund G. Robinson
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W. Leigh Smith
Lucien Yeomans

1920
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P. S. Lyon
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1921
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James Stokley
Haakon Styri
Edward R. Weidlein
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1922
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1923
C. S. Chrisman
William Dubilier
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Howard S. Levy
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Richard H. Ranger
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Edwin G. Sagebeer
Coleman Sellers, 3rd
H. Birchard Taylor
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1924
Peter Abrams
H. Carl Albrecht
Clement Starr Brinton
Karl K. Darrow
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William G. Ellis
T. R. Harrison
Valentine Hiergesell
Karl F. Oerlein
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Alfred O. Tate
Walter C. Wagner
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1925
Jack G. Binswanger
William L. Brown, 3rd
Marion Eppley
John Graham Foley
J. V. Giesler
C. R. Kraus
J. Kenneth W. Macalpine
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1926
Charles B. Bazzoni
Neils Bohr
O. E. Buckley
William Findlay Downs
Henry C. Evans
Edward L. Forstall
Lewis H. Hendrixson
Charles S. Leopold
Alfred L. Loomis
J. H. Manning
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Clement B. Newbold
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Charles S. Redding
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1927
William Adam
T. K. Cleveland
Milton W. Deisley
William DeKrafft
Hiram B. Ely
Leslie Griscom
John W. Harsch
Alfred Iddles
John Van Gasken Postles
Philip Sporn

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1928

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Simon Kimmelman, M.D. '49
William S. Monroe '25
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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

HUTH, JOHN H.: Mixed Boundary Value Problems in Potential Theory.

SAELMAN, B.: Some Formulas for Large Deflections of Beam Columns.

TOMOVICH, RAJKO: A Versatile Electronic Function Generator.

POWLEY, HOMER S.: Rapid Estimation Method for Long-Pointed Projectile Trajectories.

KAISER, H. F.: Microtrons (Electron Cyclotrons) for X and K Band Operation.

MOON, PARRY AND DOMINA EBERLE SPENCER: Interpretation of the Ampere Experiments.

HUFFMAN, DAVID A.: The Synthesis of Sequential Switching Circuits.

ALBRECHT, H. O. AND C. E. MANDEVILLE: The Storage of Energy in Some Activated Alkali Halide Phosphors.

PLAN FOR THE CELEBRATION OF THE 250TH ANNIVERSARY OF THE BIRTH OF BENJAMIN FRANKLIN

January 17th, 1956

The Celebration of the 250th Anniversary of the Birth of Benjamin Franklin will be observed by a world-wide program of communication of ideas to help improve international understanding.

Approximately 200 Societies, Institutions, Businesses, Associations and Public Service Units with a Franklin tradition have accepted the invitation of The Franklin Institute to work together in one Committee of Sponsors.

Each Sponsor will plan its own program in honor of Franklin, in its own country and in its branches in hundreds of cities. The highlights of these individual programs will be offered *on a merit basis* to the newspapers, magazines, radio, television and pictures of all countries—so that many millions of people, in their own homes, may share in the Celebration.

Theme

As scientist, inventor, author, editor, printer, philosopher, statesman and public servant, Franklin's service to man covered a wide range of activities.

In their individual celebrations, many of the Sponsors will pay tribute to Franklin in their own fields and in their traditional procedures and policies.

All have been requested by the Committee, wherever practical, to have a part of their program emphasize Franklin's great dream of a cordial understanding and cooperation among the peoples of all nations

"We may make these times better, if
we bestir ourselves."—Poor Richard

The Committee of Sponsors will develop this theme in a practical, non-political manner. Franklin himself wrote: "Would you persuade, speak of interest, not of reason."

In recognition of this philosophy, the Sponsors will tell, in their own fields, of the improvements in standards of living, health and happiness that could be achieved if better understanding between nations permitted more of the earth's rich resources to be devoted to the benefit of mankind.

They will speak of great new scientific inventions, now in the laboratories of all countries and which could benefit the people more rapidly under better conditions of international cooperation; of the challenging future of atomic energy once it may be harnessed for the service of man; of the miraculous strides of medicine which might save many lives that are now lost needlessly; of the progress in agriculture which could help to meet the needs of rapidly growing populations and reduce the haunting fear of famines; of the future aims in education which could aid so many more people to enjoy the fruits of progress; and of the philosophy of Franklin himself, who tirelessly fought for respect and equality among all peoples—to the end that, some day, there might be a deeper and more enduring kinship among mankind.

A special feature of the Celebration will be the exchange of papers among the famous scientific societies of which Franklin himself was a member during his lifetime. There were 24 such societies in England, France, Scotland, Germany, Holland, Italy, Spain, Russia and America, and more than half of them are still in existence.

The fields of activity in which Franklin made contributions and which are being covered in the Celebration include Electricity, Printing and the Graphic Arts, Publishing, the Mechanic Arts, Medicine, Advertising and Communications, International Relations, Insurance and Finance, City Planning and the Public Service, Agriculture and Botany, Religion and the Humanities, Music and Entertainment, Meteorology and Navigation, Astronomy and the Natural Sciences.

The Franklin Institute, through its Committee on the 250th Anniversary, is serving to organize the Sponsors and provide information to the press and public. It will schedule the Annual Medal Day Exercises during Anniversary Week with a program devoted to the contributions which science may make to better international understanding.

The JOURNAL OF THE FRANKLIN INSTITUTE will publish a 250-page edition on the 250th Anniversary, including many of the papers prepared by the Sponsoring Societies.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Monday, Tuesday, Friday and Saturday; 2 P.M. until 10 P.M. on Wednesday and Thursday.

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The Franklin Institute, a privately endowed institution which exists today only because of the generous benefactions of its supporters, is justly proud of the dividends in human benefits accrued from the investments made through the years by its friends. The accumulation of many bequests for general endowment and income to be used where it will do the most good, is the most important and pressing need of The Franklin Institute today.

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Additional information on gift and memorial opportunities may be obtained through the above address.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

DEFORMATION OF ROTATING BANDS ON ARTILLERY PROJECTILES*

BY

ROBERT GOOD

Plastic deformation of metals at high strain rates has for several years been the subject of a study made by the Engineering Physics Section of the Chemistry and Physics Division.[1] This study has been focussed on projectile's rotating band, which undergoes this type of deformation when it is engraved by the rifling in a gun barrel. Unfortunately, the band is hidden from view during the process so that its deformation can not be measured as it occurs. Further, the strains are too severe to use ordinary methods of strain measurement. Therefore, recourse is made to changing the physical conditions of the deformation without changing the basic considerations. A method (1, 2) has been developed for depicting the surface strains on the band during quasi-static engraving; this method might also be used in actual firing tests.

The plastic strains of a band may be determined from the relative displacements of points in or on that band. An accurately-spaced rectangular grid easily identifies many points when fixed onto the band. Then a comparison of the deformed gridwork after straining to the initial gridwork yields data for strain calculations (3). Bands have been engraved with grids on their surface by pushing them through a barrel slowly with a Universal Testing Machine.

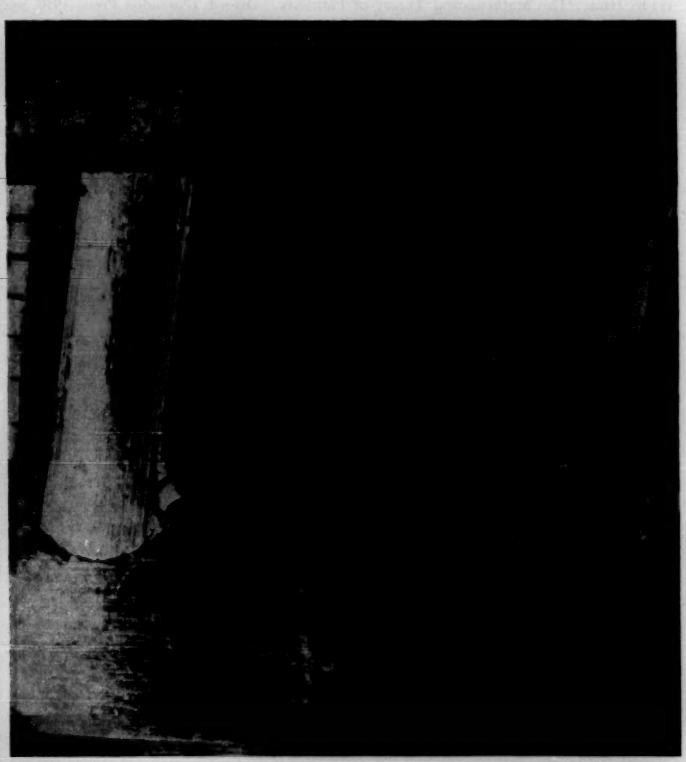
The photograph shows a metal rotating band on a small-caliber projectile that was pushed slowly (1 in./min.) through a gun barrel section. One set of lines of the grid work was scored on the band with a straight knurling tool to a depth of 0.004 in. parallel to the shell axis. The perpendicular set of lines was cut with a sharp-pointed tool to the same depth. After machining the bands, the projectiles were heated to 700° C. for one hour and quenched in oil to make the bands dead soft.

The figure shows two regions of the band: the smooth portions were made by the rifling lands because the stress was severe enough to smooth out the original grid work. The other portions show the deformed grid work because the stress under the rifling grooves was smaller. The

* This work has been supported by the Office of the Chief of Ordnance, United States Army, under the technical supervision of Watertown Arsenal Laboratories.

[1] This work has since been transferred to the Friction and Lubrication Section of the Mechanical Engineering Division.

transition area between the above regions indicates that the metal is compressed radially before it flows rearward. The tangential lines extending into the grooves indicate this flow direction. The scallop on the band rear edge results then from the axial flow. Thus the actual flow under the lands is a combination of radial and axial flow.



Rotating band on artillery-type projectile after being pushed through barrel.
(Note: Projectile Nose Up)

Projectiles with gridworks on their bands have been fired at low muzzle velocity and recovered in sawdust. The deformation of the band was essentially the same as shown in the photograph, except that the gridwork was not erased under the rifling land. This would indicate that, during firing, the body of the band flows under compression, but

the surface does not have time to be smoothed out as in the push test. The gross effect is the same on the body of the band; that is, the push test becomes valid because the basic consideration is not altered.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, EXECUTIVE DIRECTOR

PLATINUM

As 1953 was the 150th anniversary of the recognition of platinum it was appropriate for the Museum to celebrate it with the opening of an exhibit which shows the historic development and the modern applications of platinum and its related metals. This exhibit, sponsored by the International Nickel Company, Inc., is attractive enough to be well worthy of the glamorous metal it features.

When the Spaniards arrived on this continent platinum was found to be very little used by the South American Indians to whom it was known. In their hectic search for gold, the newcomers came upon it mixed with platinum, but they were unable to separate the two metals. Later, when a means employing mercury enabled them to effect a separation, the platinum was thrown away as useless. Some gold mines were actually abandoned because the gold was too "debased" with the whiter metal, to which the name *platina*, meaning silver of low value, was given.

There the matter remained until the early decades of the eighteenth century, when some of the metal was brought to Europe. At that time only seven metals were known. The platina instantly aroused the curiosity of scientific men who, while recognizing it as an eighth metal because of its weight, were baffled by its infusibility and inertness. Throughout the century it remained an enigma. Then, in 1800, William Hyde Wollaston entered into partnership with Smithson Tennant for the purpose of purchasing platina in England (at the price of 3 shillings an ounce!) and making it into industrial apparatus. Unfortunately, there was no known method for melting platinum, but Wollaston overcame this difficulty by devising a classic method for producing ingots from a wet mud of platinum powder. However, these ingots were not to be despised, for Wollaston constructed from them several boilers for the concentration of sulfuric acid. The two partners were not satisfied with this achievement, but gave the perplexing metal an intensive investigation, which had fruitful results. Wollaston discovered that platina, far from being a single metal was a composite of several. He isolated palladium, and rhodium, while Tennant went on to isolate iridium and osmium. Of the six associated metals, only ruthenium escaped their attention.

It might be questioned why Philadelphia should have been chosen as the site for any celebration related to these metals, but the city is actually very closely identified with its history. Joseph Cloud, working in the old Philadelphia Mint, isolated palladium from Brazilian gold and

put an end to the controversy which had raged over Wollaston's claim to have recovered it from Colombian platina. Then Eric Bollmann is believed to have made the first American application of platinum when he constructed a boiler for concentrating sulfuric acid for John Harrison, of the city. Finally, Robert Hare, the famous professor of chemistry at the University of Pennsylvania, improved his oxy-hydrogen blowpipe to the stage where, in 1835, he was able to melt a 25 ounce mass of platinum. Thereafter, with platinum made malleable, wider horizons opened. Extensive applications have been found in industry.

Amused visitors at one of the twenty dioramas in the exhibit "Platinum Metals—man's versatile servants."

These applications are graphically shown in a series of dioramas in the new exhibit. Telephones use millions of tiny palladium points in their relays. Rayon is made by extruding viscose through platinum-rhodium spinnerettes. Three of these metals enter into the making of a fountain pen point to enable it to give smooth continuous service. As a catalyst for the production of high octane gasoline, platinum is entering upon a great future. There are numerous other useful applications besides the most familiar one, its use in jewelry.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

THE ALPHA FORM OF 2-DESOXY-D-GLUCOSE

BY

FRANCIS B. CRAMER

The first crystalline 2-desoxy-D-glucose (2DG) was reported in 1922 by Bergmann and coworkers (1). A year later (2), the same authors described "alpha" and beta anomeric forms of 2DG. Mutarotation was so rapid in water that both forms showed the equilibrium value of +46.6° when first observed 5 min. after mixing. In pyridine, mutarotation extended over a 24-hr. period. The initial specific rotations in this solvent were +15.0° for the beta form and +90.1° for the "alpha" form. The latter value was also the mutarotational equilibrium rotation of 2DG in pyridine. Bergmann (perhaps significantly) failed to report a melting point for the "alpha" form, and recognized that it probably still contained a "certain amount" of the beta anomer.

As the result of a fortuitous occurrence it is now possible to state that Bergmann's "alpha" form contained at least 47 per cent of the beta anomer. A small quantity of 2DG was prepared, purified as the benzyl-phenyl hydrazone, and brought to crystallization essentially as described by Bergmann (1). This material was used to seed another small preparation which yielded 2 gm. of material having an initial rotation in pyridine of +150° and which mutarotated downward to +90.4° in 20 hr. The equilibrium rotation in water was +47.0° and the melting point of the crude material was 132–146° C. (uncor.).

A larger batch (8 gm. crude material) was then prepared, and purified by dissolving in water, filtering with charcoal, evaporating the solution to a syrup, dissolving in isopropanol and evaporating the solvent in vacuum to remove water. The residual syrup was dissolved in 25 ml. isopropanol and allowed to crystallize spontaneously. The mother liquor was decanted and the residual crystals (adherent to the glass) rinsed with 5 ml. isopropanol. The 2DG was then collected on a filter, washed with 10 ml. isopropanol, and dried in a vacuum desiccator. Yield, 5.5 gm. of white solid, which softened at 132° C. and melted at 134–6° C. (uncor.). The mutarotation of this material in pyridine (solvent dried over NaOH and freshly distilled) was measured: (0.0983 gm. 2DG, 11.00 ml. solution, 2.00 dm. tube).

Elapsed Time (hours: minutes)	Observed Rotation	$[\alpha]_D^{25}$
0:04⅓	+2.79°	+156°
1:05	2.75	154
4:32	2.57	144
7:20	2.43	136
27:33	1.99	111
50:10	1.89	106
97:05	1.84	103
120:20	1.84	103

It is believed that the above data represent the properties of fairly pure alpha-2-desoxy-D-glucose. Unfortunately the next crystallization of 2DG yielded a mixture of both anomeric forms showing an initial specific rotation in pyridine of +68° and an upward mutarotation. All subsequent crystallizations of 2DG in this laboratory have also yielded mixtures.

Many solvent combinations and temperatures have been tried in an effort to again obtain the alpha form. The crystalline 2DG having the highest initial rotation in pyridine was obtained as follows: 3.0 gm. of mixed alpha and beta 2DG were dissolved in 10 ml. dry pyridine and held at 100° C. for 30 min. The solution was then cooled rapidly to about 35° C. and poured at once into 30 ml. of glacial acetic acid at room temperature. The solution was then seeded with a trace of the pure alpha 2DG. Crystallization failed to take place at room temperature but progressed slowly in the refrigerator. After 44 days, the solid was collected on a filter and washed with acetic acid and acetone. The initial rotation in pyridine was +145° and mutarotation was downward. This material evidently contained at least 7.8 per cent of the beta anomer.

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BOOK REVIEWS

SAMPLE SURVEY METHODS AND THEORY, by Morris H. Hausen, William N. Hurwitz and William G. Madow. New York, John Wiley & Sons, Inc., 1953.

VOLUME I, METHODS AND APPLICATIONS. 638 pages, 16 × 24 cm. Price, $8.00.

VOLUME II, THEORY. 332 pages, 16 × 24 cm. Price, $7.00.

The great Gallup Fiasco of 1948 and two bestsellers by Kinsey have aroused the public's interest in statistical studies and predictions made from sample surveys. How can the layman be expected to discern between facts and fancy if even the experts get trapped occasionally? To the aid of both come now the above two volumes. Their authors are well qualified and can speak with authority. Their contributions to the field have been numerous; their experience stems from years of work at the U. S. Bureau of the Census.

Sampling surveys are really a rather commonplace experience. Our every movement and decision depend on the subconsciously applied theory of sampling. Every observation and every opinion formed are representative of a sample only. No one knows all facts or, in the words of Laplace, he could predict the course of the universe. But are these personal observations reliable? Are the decisions made wisely? Social scientists, psychologists, economists, and business men alike are constantly beset by sampling problems. During the past years there has emerged a crystallized mathematical structure and the present two volumes mark a milestone in the rapidly maturing field.

The authors aim to present both practice and theory in a form which is suitable as reference or textbook. In order to fulfill this aim, the first volume discusses problems and practice of survey sampling while the second deals with the theory. Naturally, there is some duplication in the coverage: chapters four through eleven in both volumes deal separately with the two aspects of like topics. In notation and terminology, the second volume is addressed to the statistically trained reader, while the first is more or less expository. But it is the intention of the authors that both volumes stand and serve their respective purposes independently.

The first volume contains twelve chapters. An introduction to sampling principles is followed by an exposition of two important factors, bias and non-sampling errors. The third chapter deals with sample designs for symmetric and skewed populations. The next four chapters discuss formal sampling procedures: simple and stratified random or cluster sampling. The latter problem introduces, of course, the cost of the survey. The thorough treatment of cost functions and their construction is rather noteworthy. Optimum designs are discussed even for the more complex cases. The next two chapters draw our attention to the problem of the primary sampling unit or PSU. The effect of the size of the PSU on sample parameters is stressed at this point through variations observable in averages, ratios, and sampling fractions. The tenth chapter delves more deeply into the problem of population variances estimated from sample returns. The eleventh chapter proposes some methodological ramifications useful to obtain efficient designs in certain special cases. The final chapter presents five interesting case studies: retail stores, population surveys, surveys of manufactures, cluster samples of persons and dwelling units, and quality control methods.

The second volume presents a comprehensive view of the basic theory. Its first three chapters acquaint us with nomenclature, probability theory, and statistical sampling parameters. The next eight chapters agree in scope and title with the corresponding chapters of the first volume. They develop a firm theoretical foundation for the concepts and theorems which were given without proof in the first volume. Lagrangians are used to obtain optimum conditions for the various cost functions and sampling plans. The final chapter treats a mathematical model for

response errors in social surveys; it is primarily concerned with errors in the data collection.

An outstanding feature of both volumes is the large number of formulas that apply to a variety of practical and important situations. Once the reader has fully grasped the notation he will find that these formulas cover more than the ordinary number of cases. This fact makes the two volumes at once valuable references and comprehensive texts. The well chosen problem illustrations are taken from the files of the U. S. Bureau of the Census. Examples are properly graded, although not always numerous enough. Each chapter ends with a list of references from the American and British literature.

Theorems in both volumes are couched in the language of probability sample statements for which objective measurements of the sampling errors can be made. The precision of designs and sample results can thus be evaluated. At first reading, one may find it somewhat difficult to assimilate notation and definitions. Therefore, it is regrettable that neither volume contains a glossary of terms and symbols. The serious student might do well to prepare such a list for his own use. On the whole, both volumes reflect the position of leadership that the U. S. Bureau of the Census and the authors have assumed in this field. *Sample Surveys* is a fine reference for practicing statisticians and a good text that teachers and students will welcome. It will aid those who must rely on sample information in their understanding of statistical inference. Finally, it is hoped that somehow these two volumes will also fall into the hands of those who continue to abuse the public by their ignorant or maliciously deceptive use of statistical terms—and help clean up radio, television, politics, and polls.

CARL HAMMER

THUNDERSTORM ELECTRICITY, edited by Horace R. Byers. 344 pages, illustrations, 18 × 25 cm. Chicago, University of Chicago Press, 1953. Price, $6.00.

Thunderstorms and their accompanying lightning have been studied because of their destructive and nuisance effects. Too little attention has been given to the part they play in the balance of nature. In the study of plant nutrition one becomes aware of the importance of lightning in the nitrogen cycle of the soil and the biosphere, but little has been accomplished in the geophysical sciences where interest centers on the part played by thunderstorms in the balance of the atmosphere. In the more than two hundred years that have passed since Franklin's experiments, many theories of the mechanics of charge separation in thunderclouds have been advanced. Few of them have fitted the facts later discovered. Today we still have no generally accepted theory, but we believe a great deal more is known of the processes which can cause the separation.

In 1950 a conference was held in Chicago under the auspices of the U. S. Air Force through its Geophysical Research Directorate, to which a group of active workers from several lands were invited. The outcome of this conference has been the filling of some gaps in our knowledge of thunderstorms. The papers representing the summaries of what these workers had learned in their special field have now been published in book form, having had only a limited distribution. The resultant volume offers an excellent survey of the field, and is likely to remain the most acceptable textbook on the subject for some time to come.

The book is arranged so that the theoretical aspects are first treated and the practical applications are treated later. Although physicists and meteorologists will find the material presented to be invaluable, the book will appeal to other students who are not easily discouraged by theories and mathematical treatment. Those readers whose interest in thunderstorms is of a more practical nature will be rewarded in those studies relating to effects upon aircraft, communication systems, and power lines. In every instance the subject has been presented by a writer of acknowledged authority in the field.

The book is well illustrated, and each chapter has an adequate bibliography. It is of very high worth, but the absence of an index is an inexcusable defect. T. C.

COMMUNICATION THEORY, edited by Willis Jackson. 532 pages, diagrams, 16 × 26 cm. New York, Academic Press, Inc., 1953. Price, $11.00.

The appearance in 1948 of Shannon's papers on the mathematical theory of communica-

tion has stimulated a great deal of activity in this field. The book under review is a collection of papers read at a symposium on the subject held under the auspices of The Institution of Electrical Engineers, London, September 22–26, 1952.

Communication theory, or information theory as it is more commonly and less precisely referred to, is concerned with a message source, communication channel and receiver. Shannon has developed measures for the information content of the message source, for the information-handling capacity of the communication channel and the receiver. The most successful application of the theory has been to the analysis of codes and modulation systems which make it possible to utilize the channel capacity more efficiently, and also make it possible to overcome the effects of noise or errors in the system. However, the principal effect of Shannon's work has been to show that probability theory and statistical methods are powerful tools in handling communication problems. As a matter of fact, most of the papers read at this symposium are concerned with the use of a statistical approach to phases of the communication problem, while only a small portion of the papers depend on Shannon's theory.

The symposium consists of 38 papers and a concluding discussion. The first paper contains a fine summary of Shannon's theory. This is followed by a series of papers on coding, combatting noise in a communication system by coding, and analyses of various modulation systems. There is a set of papers on the application of statistical methods to the detection of signals in the presence of noise. A series of papers is devoted to determining the essential elements of pictures and the application of the results to the problem of compression of television transmission. Several papers are devoted to the study of spoken language and its structure, the nature of hearing, and how to design machines for synthetic speech and hearing.

Shannon's theory is concerned with the physical and statistical properties of messages and not their meaning. However, one of the papers in this collection deals with the extension of communication theory to include the semantic content of messages.

H. PLATZER

THEORY OF FUNCTIONS OF REAL VARIABLES, by Henry P. Thielman. 209 pages, 15 × 22 cm. New York, Prentice-Hall, Inc., 1953. Price, $5.00.

The author, who is a Professor of Mathematics at the Iowa State College, set himself the twofold task to familiarize the student with the fundamental concepts and methods of the theory of functions of real variables and to introduce him to the postulational approach to various mathematical theories. The topics considered are those which are generally included in standard mathematics courses for first-year graduate students and advanced undergraduates.

This book has several merits. In addition to material covered by older texts, it includes several early chapters devoted to the theory of sets, to topological spaces without and with a metric, and to the theory of measure. The order in which the topics are presented is excellent from the didactic point of view; whenever possible, without interrupting the main line of development, new concepts are introduced as generalizations of ideas already familiar to the reader. The exposition of the largest part of the material in postulational form results in a large economy of space. The author's explanatory remarks, definitions, statements of the theorems and their proofs are remarkably concise and lucid.

Many existence proofs are relegated to Problems for the reader, interspersed in the text. Occasional examples are used to illustrate a difficult point.

In all, the author has been very successful in presenting on 203 pages a wealth of material in a well integrated and readily assimilated form. The present writer recommends it both as a textbook for a course and as a first introduction to the subject matter for the studious general reader.

OTTO R. SPIES

ULTRA HIGH FREQUENCY PROPAGATION, by Henry R. Reed and Carl M. Russell. 562 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1953. Price, $9.50.

Ultra High Frequency Propagation by Reed and Russell is an example of a new type of reference book which is now making its appearance and which, incidentally, has gladdened the hearts of practicing engineers. Books on radio frequency propagation a dec-

ade ago were primarily classical texts dealing exclusively with Maxwell's equations and similar mathematical presentations of physical phenomena. The newer books, however, are written in a manner which appeals more to the systems engineer. The foremost example of this type of writing is found in the M.I.T. Radiation Laboratory Series which gained such wide acceptance following World War II and is still universally used by engineers in this field.

The systems approach to a subject such as ultra high frequency propagation does not imply that the text will be weaker in fundamentals or less exacting in its formal treatment of physics. It does mean, however, that the volume will contain a large amount of empirical data and information which necessarily supplements the classical theory. Some space is usually given to explaining how the empirical data were obtained and how this information may be applied to engineering problems. This type of treatment admittedly results in a more lengthy volume or limits the book to a narrowly specialized area of coverage.

In the search for an appropriate title for a book of this kind the authors frequently encounter some difficulty. Reed and Russell have covered a great deal of territory under the heading, "Ultra High Frequency Propagation." Among communications engineers, ultra high frequency applies specifically to the range of 300 to 3000 megacycles. However, this work covers a much broader range, including VHF and the microwave region. Much attention is given to antennas and particular operational considerations, since these have a large effect upon the system aspects of a propagation study.

A great deal of work has been done since World War II in an attempt to better understand the factors which influence high frequency propagation all the way from the local disturbances in the induction field to the far reaches of the diffraction region. The book considers these factors in relation to a variety of propagation arrangements including ground-to-air and ground-to-ground. The effects of polarization and meteorological conditions are treated more thoroughly than in any book so far published.

Reed and Russell have prepared a very acceptable reference volume which should be useful to engineers and others confronted with propagation problems such as those found in radar and various radio communication links. It is certain that the authors' facility in dealing with their subject results partly from teaching experience as well as their close association with the timely experimental programs being conducted at the Naval Air Test Center on aircraft and radio systems. In fact, it is information such as has been gathered here that will make it possible for engineers to squeeze the narrowing margin of performance improvement from future radar and high frequency communication systems.

C. W. HARGENS

CONQUEST OF THE MOON, by Wernher von Braun, Fred L. Whipple, and Willy Ley. 126 pages, color plates, 21 × 28 cm. New York, The Viking Press, Inc., 1953. Price, $4.50.

The sequel to *Across the Space Frontier* has been published as *Conquest of the Moon*. Again some of the same authoritative writers, Wernher von Braun, Fred L. Whipple and Willy Ley, who achieved such distinction in their previous work, have collaborated in painting a picture of an expedition which is assembled, travels to, lands, explores and leaves the moon to return to the space station. Again the same skillful touch of a literary craftsman, Cornelius Ryan, collates and edits the material to make the reading almost an experience in space travel. The same sensational artists, Chesley Bonestell, Fred Freeman and Rolf Klep, illustrate the text in inimitable fashion.

The description of the vehicles and the planning indicates the grandiose thinking of von Braun while the astronomical intricacies are handled in a neat fashion by Dr. Whipple. This reviewer could find only one error in the text and that concerns the minimum distance of Venus. It is not 20 million miles but 26 million miles.

The description of the quarters, the provisioning, the type of life aboard the vehicles is covered by Willy Ley. There is a division of task that is apparent, yet a coherence to the entire book which is ever present. The ingenuity woven into this exploration by the authors is so real it appears almost as the account of an expedition already made rather than a projected one in the distant future.

The book is magnificently written but it

does not have as true a ring as the previous work. When the conquest of the moon is attempted man will have been in a space station for a long time. During this time the hazards of space will have been explored and the excessive precautions ennumerated in the present work may be unnecessary on the moon in that distant day.

The story is one not concerning the first trip to the moon, but a much later one. It is inconceivable that the first trip to the moon will be a manned one. The first will, in all probability be an unmanned journey around the moon and back to the space station. This will be followed by another expedition of two or three intrepid explorers who will touch down on the moon for a very short interval to lay the groundwork for still another expedition of the type described in this book.

Conquest of the Moon is a must for those who have any interest whatsoever in breaking through the last frontier—that of space.

I. M. LEVITT

PRINCIPLES OF TRANSISTOR CIRCUITS, edited by Richard F. Shea. 535 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $11.00.

The transistor, having passed through the exciting, tempestuous and sometimes disappointing history of a child prodigy, has now come of age and is to be accepted as a working, albeit distinctly individual, member of our technologic team. The evidence of the transition is the publication of this first text on the engineering aspects of transistor circuits. Of course, the transistor, although now in its majority, has not attained its full stature; its growth is still rapid. The authors and publishers of this volume are to be commended on their courage in their decision to publish now a volume which may suffer an obsolescence more rapid than normal. We hope this same courage will lead them to prepare the revisions necessary to keep abreast of the progress yet to come.

The arrangement of the book is reasonable, much the same as one would anticipate for a parallel volume *Principles of Vacuum Tube Circuits*. First we are given an elementary discussion of the semi-conductor principles which are important to the functioning of the transistor, and their use in describing the inner workings of transistors over ranges of temperature, power and frequency. This is followed by a brief chapter on physical forms, construction, and graphical characteristics. The third chapter gives the engineer's working tool, the equivalent circuit, in considerable detail, for the various possible connections. In the balance of the book, this basic material is put to good use in elaboration of the operating principles of all the various circuits in which the device has been found useful.

The book is singularly up-to-date, and yet shows no evidence of being thrown together hastily. In spite of the large number of contributing authors, there is no unevenness of level or style, which bespeaks careful editing. It is very complete, introducing matrix methods where useful, treating feedback principles clearly. Distinctions are drawn between the transistor and the vacuum tube, emphasizing the differences in circuit design for the two distinct devices and making clear that the transistor is not just a small triode without a heater.

The book should make an excellent college text in a course which would serve to teach more than just transistor circuits. Its first chapter may need supplementary material, which can easily be found in Shockley's *Electrons and Holes in Semi-Conductors*. The two volumes form a fine pair for the engineer interested in using transistors.

We recommend a reading of the preface to this volume; it serves as an excellent review of the book.

ALAN C. BYERS

EXPERIMENTAL NUCLEAR PHYSICS, VOLUME II, edited by E. Segrè. 600 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $12.00.

This is the second volume in the series of three books aimed at bringing the experimentalist up-to-date. Part VI, "A Survey of Nuclear Reactions," was written by Philip Morrison and Part VII, "The Neutron," was written by Bernard T. Feld. As in the first volume, the emphasis is upon fundamentals and the understanding of basic phenomena and basic theories. This series has been quite successful in achieving its aim and in doing so, has produced a standard reference useful also to the non-nuclear scientists.

Both authors present this material in a pleasing, easily readable, unhurried style.

Terms, even the most elementary, are clearly defined. Derived formulae are carefully interpreted and the limitations of various theories are clearly indicated. Theory is presented with a minimum of mathematics and is carefully correlated with available data. A great deal of experimental data is given in the 119 figures and the 49 tables in the text. There are almost 800 references to the published literature and several to recent unpublished material. Subject and author indices for the combined parts are given.

Dr. Morrison has divided his material on nuclear reactions into eleven sections: (1) The Conservation Laws, (2) The Data of Nuclear Reactions, (3) The Nuclear Model, (4) Nuclear Level Widths, (5) The Course of Nuclear Reactions, (6) The Dispersion Theory: Resonance Reactions, (7) Some Typical Nuclear Reactions, (8) The Deuteron as a Projectile, (9) Radioactive Processes in Nuclear Reactions, (10) Nuclear Fission, and (11) Nuclear Reactions at High Energy. This last section is the longest one and is indicative of the increasing experimentation with high energy accelerators. Nuclear theory for high energies is quite different from the low energy theory using the compound nucleus idea given in section 6, since processes non-existent at low energies become important in BEV range. There is only a cursory mention of mesons, but there is the warning that our present theories are still only in the formation stage. It is interesting to note that Dr. Morrison, in introducing the subject of nuclear fission, makes the statement "All information here presented, without exception, is from the published literature."

Dr. Feld presents the properties of the neutron in five sections: (1) Properties and Fundamental Interactions, (2) Interactions with Nuclei, (3) Sources and Detectors: Neutron Spectroscopy, (4) Interactions of Neutrons with Mattet in Bulk, and (5) Coherent Scattering Phenomena with Slow Neutrons. Section 3 is the only frankly experimental discussion in the book. However, even here, apparatus and techniques are explained from first principles. Section 4 covers elementary pile theory, while Section 5 discusses neutron diffraction. It is clear that the neutron has become a powerful tool in nuclear physics since its freedom from a net electrostatic charge limits its interactions to nuclear and magnetic ones. There is no great field of proton physics.

It is difficult to estimate the contribution of the editor to these volumes, but with such uniformly excellent results, one feels compelled to believe that Dr. Segrè's help was of considerable value. One cannot help but be grateful for presentation of detailed discussions on fundamentals not easily found elsewhere. An example of this is the discussion of the possibility of a net charge on the neutron. These volumes should be of great value to many, and this includes undergraduates studying nuclear physics.

LEONARD MULDAWER

EXTERIOR BALLISTICS, by E. J. McShane, J. L. Kelly and F. V. Reno. 834 pages, 15 × 22 cm. Denver, University of Denver Press, 1953. Price, $12.00.

The topic of Exterior Ballistics is a difficult one to treat due to its dependence on mathematics and physics, and due to Government security regulations. The latter results in most literature in this field, including ballistics tables, being classified at least "restricted." All too frequently such tables are unaccompanied by explanations of any sort as to how they are made. They have been taken on faith and made the basis of instrument design in cases where not strictly applicable. Any addition to the scant unclassified literature in this field is welcomed by those engaged in it. These people are generally working on government projects and have access to the classified literature as well.

This lithoprinted tome treats the mathematics and physics involved as such before discussing the trajectory equations and methods of solving them. It does not include any extensive ballistic or firing tables, although general reference is made to some of them. The references to other literature are scant but interior references from one chapter and section to another are both adequate and easily found through the numbering of chapter and section on each spread.

The chapter on "Bombing" should be more correctly labeled "Horizontal Bombing," since it is limited to level bombing and assumes the presence of a bombardier. Toss bombing is not treated, but dive bombing is mentioned briefly in the chapter on approximate methods used in order to obtain solutions of the trajectory equations.

The manuscript for this book covers work done by the authors near the time of the Second World War and was brought to essentially its present form in 1949. As a result the latest form in which some gunnery tables have been prepared is not discussed. The presentation might have been improved by the inclusion of more excerpts from ballistic or firing tables with specific statements as to how they were made. D. B. HOUGHTON

INFORMATION THEORY, by Stanford Goldman. 385 pages, diagrams, 15 × 22 cm. New York, Prentice-Hall, Inc., 1953. Price, $9.00.

Since the appearance of Shannon's fundamental work in communication theory, an increasing number of engineering schools are giving courses in this field. Hence there is a need for a textbook which presents the material in a form more palatable to the engineering student than the elegant style of Shannon's papers. This book fills the need for a lucid treatment on a fairly modest level.

Goldman begins his book by discussing the information theory of discrete systems. He defines what he calls "semantic" information and "language" information. Roughly speaking, the measure of semantic information is obtained from the probabilities of events, while the measure of language information is obtained from the probabilities of the symbols used to describe events. Language information turns out to be the same thing that is talked about in conventional communication theory, while semantic information is something new. It is the new thing that makes it proper for Goldman to call his book *Information Theory*, while Shannon calls his *Communication Theory*. Thus language information theory is more popularly referred to as communication theory, while semantic information theory is information theory. However, Goldman's book contains very little development of semantic information theory except for its definition and a rather complete exposition of communication theory.

There is a thorough discussion on sampling theorems in the time and frequency domains. These theorems make it possible to apply the results of information theory for discrete systems to continuous systems of signals limited in bandwidth and duration. Furthermore, the sampling theorems form the basis of pulse modulation systems. Hence a study of modulation systems, their information capacities and noise reducing properties is carried out with care and clarity.

The book contains a discussion of Wiener's theory of prediction and filtering. The approach used is an elaboration of the heuristic method of Bode and Shannon, rather than the rigorous approach of Wiener.

The concluding chapter contains a summary of the subject as well as a discussion of open problems in information theory.

This reader has a few quibbles: (1) the consistent use of "discreet" where "discrete" is usually used; (2) the great abundance of footnotes, most of which could easily be included in the text (it is frequently necessary to interrupt a train of thought to read an elaboration of an idea in the footnotes); (3) the use of unconventional symbols and terminology which adds precision but places a burden on the student who must learn new terminology when reading reference material. However, the book is highly recommended. It fills the need for a good expository treatment of the modern approach to communication problems.

H. PLATZER

FLIGHT INTO SPACE, by Jonathan Norton Leonard. 307 pages, 14 × 21 cm. New York, Random House, Inc., 1953. Price, $3.50.

A science editor of a large, widely read news magazine enjoys a curious and unique position. Before him, in the form of communiques from the laboratories, colleges and research establishments, come the latest developments in the vast panorama of science. By checking item by item the achievements of the past, the trend of the present and the aspirations of the future unfold into a pattern always discernible to the skilled observer.

Such an observer is Jonathan Norton Leonard, Science Editor of *Time Magazine*. In his new book, *Flight Into Space*, his intimate contact with the receding frontier of science has enabled him clearly and brilliantly to portray the present-day status of space travel. It has enabled him to analyze critically the many approaches to the departure from the earth. It permitted his assessment of the various programs which have been

proposed. It enables him to separate fact from fancy. Even the philosophy behind the supreme adventure can be delineated.

All the facets of the space travel picture are explored in the 25 chapters of the book, beginning with the rocket take-off at White Sands and then dealing with the trials and tribulations of successful rocket firings. What the lay public little realizes is the extent of the vast complexity of instruments which comprise a rocket—be that rocket a guided missile or an explorer of the atmosphere. If a single component fails, the entire firing—costing in many cases in excess of a quarter million dollars—is a sterile one. The author treats this phase with dispatch and authority.

The next step into space is a resume of von Braun's grandiose dreams of a space station. In this section, for the first time, a digest of the most significant paper of Dr. Milton Rosen of N.R.L., read at the Second Space Travel Symposium at the Hayden Planetarium in 1952, is included in a popular book. True to the sub-title, here are facts to counter fancies and no one should be exposed to the let's-do-it-now school without being shown the faults, pitfalls, shortcomings and hazards of this project. The sympathetic treatment of Dr. Rosen's arguments, to this reviewer and observer of the space travel picture, means that the detached, objective thinking of our government agencies is upheld by the opinion moulders of the press and this can delay any precipitous action which could lead to a national catastrophe.

The author's description of the human body in the totally alien and hostile environment of outer space is exposition of the first magnitude. Writing with a lucidity indicative of complete familiarity with the problem, he shows that the process of living in space is not a simple one, in fact, every second of man's existence off the earth will present a challenge both physical and psychological.

There is a section devoted to space telescopes, the solar system and finally a section on ways and means of getting around time and distance. The latter could have best been omitted. It is not in keeping with the text though it does furnish the philosophical aspect to which a portion of the book is dedicated.

Flight Into Space is the soberest work on space travel to appear and to those who want their facts without the veneer of unbounded optimism, who would like the story presented coldly, dispassionately and objectively, this work is highly recommended.

I. M. LEVITT

ESTIMATION OF ORGANIC COMPOUNDS, by F. Wild. 239 pages, 15 × 23 cm. New York, Cambridge University Press, 1953. Price, $5.00.

This rather short volume of 239 pages describing a number of relatively simple methods for determining functional groups in organic compounds is primarily a textbook for the student or one who has been away from analytical procedures for a long time. A variety of simple reactions which depend on some characteristic property of the test compound are listed. Many of these tests are familiar to the industrial chemist. Descriptions of the specific apparatus required in these tests are given in detail. Electrono-metric titrations, colorimetric methods, use of the refractometer, and several of the other tools of the analytical chemist are described.

Both macro and micro methods for estimating the groups concerned are given in textbook detail. Typical groups that are discussed are Olefines, hydroxyl groups, mercaptans, carbonyl compounds, amines and amino groups, nitro, cyano and nitroso compounds, and others. It is essentially a laboratory manual offering ready access to simple tests for common groups in organic compounds.

S. N. MUCHNICK

BOOK NOTES

METALLURGICAL DICTIONARY, by J. G. Henderson, assisted by J. M. Bates. 396 pages, 16 × 24 cm. New York, Reinhold Publishing Corp., 1953. Price, $8.50.

Containing over 5000 definitions and descriptions covering all the most essential terms in both production and physical metallurgy, this new dictionary is up-to-date and comprehensive. It fills a need in the metallurgical field, and, through its system of cross references and synonyms, will be a boon to specification writers, purchasing agents, architects and builders, as well as to metallurgists themselves.

CLIMATE, VEGETATION & MAN, by Leonard Hadlow. 288 pages, illustrations, 14 × 22 cm. New York, Philosophical Library, 1953. Price, $4.75.

The author, who is Senior Geography Master at Burnage High in Manchester, England, has set forth his ideas of the influence of climate on plant, animal and human life. The story is a fascinating one, ranging from eclipses, rainfall, icebergs, seasons and temperature zones to the rocket exploration of the upper atmosphere. For those who are interested in weather and climate, the book is recommended as a handy reference work, well illustrated with photographs and charts. For so small a book, the author has done a remarkably complete job.

KINEMATICS OF MACHINES, by Rolland T. Hinkle. 231 pages, illustrations, 16 × 24 cm. New York, Prentice-Hall, Inc., 1953. Price, $4.75.

Emphasizing the relative motions of machine parts, this compact volume covers a wide variety of mechanisms, both from a theoretical and a practical point of view. The first two chapters treat relative motion, inversion and the angular velocity theorem in some detail, so that a student with just a knowledge of calculus and no engineering mechanics can get the basic concepts needed for the rest of the chapters. The subjects covered include velocities in mechanisms, velocity polygons, acceleration, graphical and analytical methods, cams, rolling contacts, gearing, flexible connectors, gear trains, translation screws and miscellaneous mechanisms. Each chapter concludes with a set of problems (no answers given). Well illustrated and well organized, the book should prove useful in engineering mechanics.

FRESH-WATER INVERTEBRATES OF THE UNITED STATES, by Robert W. Pennak. 769 pages, diagrams, illustrations, 19 × 25 cm. New York, The Ronald Press Company, 1953. Price, $14.00.

This is the first comprehensive work on American fresh-water invertebrates since 1918. Because much new information has accumulated since then, the need for this treatise is obvious. While the author uses a minimum of original material gained from his own research, he has nevertheless done a tremendous amount of selecting and organizing material from some 5000 sources. Emphasis is on natural history, ecology and taxonomy, while details of anatomy and physiology have been minimized. The publisher is to be congratulated on the fine typography, and the author deserves a great deal of praise for his monumental work in accumulating the material for the book, which is sure to take its place on the reference shelves of biologists, zoologists, limnologists and entomologists.

PARTIAL DIFFERENTIAL EQUATIONS IN ENGINEERING PROBLEMS, by Kenneth S. Miller. 254 pages, 15 × 22 cm. New York, Prentice-Hall, Inc., 1953. Price, $4.75.

While there is nothing new in this text, the author has approached the subject of partial differential equations from a strictly engineering viewpoint. His chief criterion for judging what to include and what to leave out is the practicality of the material. Pure theory is avoided; the main purpose is to solve the necessary equations which arise in engineering work. The six chapters, each of which is followed by a list of problems, cover the following subjects: derivation of partial differential equations, Fourier series, separation of variables, the Fourier integral, Legendre, Bessel and Mathieu functions, and properties of second-order partial differential equations.

FILTER MANUAL, by Carl Bakal. 136 pages, plates, charts, 16 × 24 cm. San Francisco, Camera Craft Publishing Co., 1953. Price, $3.00 (paper).

For the amateur photographer who wants to take better pictures, through the use of filters, this manual will be of real help. Both color and black-and-white photography are treated, as are indoor, outdoor, still life, portraits, etc. Arranged in alphabetical order as to picture subject, this manual is easy to use and practical. For example, under the main heading "Distant Landscapes," there are four sections (True Rendition, Haze Penetration, Atmospheric Effects and Artificial Fog Effect), each of which recommends the correct filter to use in order to obtain the best results. Also of value are the detailed charts showing what filter to use under varying conditions of light, with different cameras and types of film.

ELEMENTARY QUANTITATIVE ANALYSIS, by Ralph L. Van Peursem and Homer C. Imes. 383 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $4.50.

Designed for use in a two-term introductory course, this text is divided into four sections: Principles, Calculations, Laboratory, and Techniques. The content, based on a national survey of quantitative analysis courses in U. S. colleges and universities, includes all material which was treated in 25 per cent or more of such courses, as well as some material of lower frequency. The presentation of subject matter has been aimed at premedical, preengineering and chemistry students, with the choice of emphasis left to the teacher for individual classes.

ORGANIC SYNTHESES, VOLUME 33, edited by Charles C. Price. 115 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $3.50.

Volume 33 of *Organic Syntheses* gives procedures for the preparation of compounds and equations for the reactions for 40 different organic compounds. Preparations are listed alphabetically under common name, with the *Chemical Abstracts* indexing name listed as a subtitle, if it differs from the common name. All procedures have been carefully checked and the subject index includes all material listed in Vols. 30–33.

The JOURNAL editors wish to call attention to the following list of titles, available through Dover Publications (New York). These are reprints of valuable scientific books, issued primarily so that students might acquire the fundamental books in science at a reasonable price. All of these are 14 × 20 cm., and all were issued during 1953.

WHAT IS SCIENCE?, by Norman Campbell. 186 pages. Price, $1.25 (paper); $2.50 (cloth).

HISTORY OF ASTRONOMY FROM THALES TO KEPLER, by J. L. E. Dreyer. 438 pages. Price, $1.95 (paper); $3.95 (cloth).

LECTURES ON CAUCHY'S PROBLEM, by Jacques Hadamard. 316 pages. Price, $1.70 (paper); $3.50 (cloth).

THE THEORY OF ELECTRONS, by H. A. Lorentz. Second edition, 343 pages. Price, $1.70 (paper); $3.50 (cloth).

ELEMENTARY MATHEMATICS FROM AN ADVANCED STANDPOINT, by Felix Klein. Volume 1, Arithmetic Algebra Analysis. 274 pages. Price, $1.50 (paper); $3.25 (cloth).

SUBSTANCE & FUNCTION *and* EINSTEIN'S THEORY OF RELATIVITY, by Ernst Cassirer. Both books bound as one. 465 pages. Price, $1.95 (paper); $3.95 (cloth).

INTRODUCTION TO SYMBOLIC LOGIC, by Susanne K. Langer. Second revised edition, 367 pages. Price, $1.60 (paper); $3.50 (cloth).

THE WORKS OF ARCHIMEDES, edited by T. L. Heath. 378 pages. Price, $1.95 (paper); $3.95 (cloth).

CURRENT TOPICS

High Speed Teleprinter.—A major innovation in printed communication, previously restricted to the Armed Forces, was made available for general use recently, by Kleinschmidt, Inc., of Deerfield, Ill.

It is a light weight, compact teleprinter designed to send and receive messages at 100 words per minute, and actually capable of speeds up to 150 w.p.m., 2½ times faster than teleprinters presently used.

The 45-lb. page printer, only about one-third the weight of those now in use and containing 30 per cent fewer moving parts, is the creation of Edward E. Kleinschmidt, one of the nation's foremost communications scientists and co-inventor of the present teletype machine.

The teleprinter, when receiving from a previously prepared perforated tape, can hit a top speed of 15 characters a second regardless of how slowly the message originally was typed by the operator. Present printers, weighing about 110 pounds, have a top rate of 60 w.p.m., approximately the average speed of a human operator.

The 78-year-old inventor, who holds the John Price Wetherill Medal of The Franklin Institute, personally gave the machine its first public demonstration. It has been used exclusively by the Army Signal Corps and other military branches since 1949.

Kleinschmidt said the teleprinter, now in mass production, is available immediately for sale to commercial users. He disclosed that his firm, which is continuing to fill government orders, is geared to turn out a teleprinter every 10 minutes. The company also will continue work on some 45 developmental projects for the government, many of them classified.

"Within a decade," Kleinschmidt said, "virtually every major phase of business and industrial operations will involve extensive use of teleprinters; everything from inventory and production control to remote operation of automatic business machines."

He asserted that even the major users of teleprinters today, those in the communications and transportation fields, "have come nowhere near realizing the full potential of the machine."

He said this is due largely to "a general public unawareness of the innumerable uses to which the machine can be put," a condition, he added, his company hopes to correct.

The teleprinter provides a system of electronic communication whereby printed messages can be transmitted simultaneously to any number of similar machines. Receipt of the message is instantaneous with the transmission, regardless of distance from the sender. Each unit is capable of both sending and receiving, and will operate on both wire and radio circuits.

Controlled by a keyboard similar to that on a standard typewriter, it can be operated by anyone able to use a typewriter. Most widely used by news wire services and the telegraph industry, the system is also in operation among railroads, airlines, the petroleum and mining industries, and other businesses where rapid and accurate communications is a major factor.

Two standard versions of the Kleinschmidt page printer are being offered. Model 150 is essentially the Signal Corps unit converted for com-

mercial use by eliminating highly specialized features which were designed for military needs. Model 155 is a further streamlined version which provides all features essential for most commercial purposes.

For ultra-rapid transmission, Kleinschmidt is producing a combination typing reperforator and tape transmitter. This machine, Model 120, overcomes the speed limitations of manual typing when used to transmit to a page printer. It also makes practical high speed remote operation of automatic business machines, addressographs, calculators, printing-adding machines, and other mechanical bookkeeping and accounting devices.

All three units have been styled for modern office use by Morton Goldsholl, noted Chicago product and industrial designer.

New Industrial X-Ray Film.—A new ultra-fine grain industrial X-ray film has been developed by the Du Pont Photo Products Department. Known as Type 510, the film can be used at low voltages (10–20 peak-kilovolts) to examine low-opacity materials, such as aluminum, magnesium, corn, and wheat. With million-volt X-ray or betatron equipment, it may be used to examine steel up to three inches in thickness.

The new film can be used to particular advantage on subjects where very minute detail is desired, since the fine grain emulsion permits detection of tiny flaws in, for example, spot-welding in aluminum sheeting. The same is true for the detection of foreign contamination in grain prior to milling, a relatively new use of X-rays.

Du Pont Very Fine Grain Industrial X-Ray film is made for radiography applications where minute detail is more important than speed. Hence the new film is only approximately one-third as fast as Du Pont Type 506 Industrial X-Ray film.

Type 510 is designed for use without calcium-tungstate intensifying screens. It is available in all standard sizes and is packaged as "NIF" non-interleaved film. "NIF" packaging eliminates the paper separators between individual pieces of film, thus simplifying the loading of holders for exposure.

The Image Amplifier in Movies and TV.—The year 1953 was a significant one in the life of the image amplifier. This was the year in which the first commercial models of this bright-image fluoroscope became available from the Westinghouse Electric Corporation. Thus marked the culmination of a decade of development, by which the image seen by the diagnostician is at least 200 times brighter than that on the conventional fluoroscope screen. Considering the many, and often disheartening, difficulties that lay along the way in the production of consistent, high-amplification tubes, the attainment of commercial models is a significant milestone in itself. Other things of general and technical interest occurred too.

The image amplifier, for one day last April, became a television star. It was the means by which thousands in their homes saw the beating of a human heart and glimpsed other organs just as the radiologist views them. Only the super-bright image, which brings out great detail, made this possible.

While this was something of a dramatic demonstration, plans are underway to put the image amplifier on television for a more serious purpose—medical-student instruction. With present extremely weak fluoroscopic images it is impossible to give class instruction from "life." The plan now under consideration is to televise the

picture produced on the screen of the image amplifier to a number of medical classroom receivers. Because this will be close-circuit television more lines will be scanned per inch—giving more detail.

The image amplifier has gone into the movies, too. It has been arranged with a special lens system and a special 16-mm. movie camera so that the doctor can record the action, say, of the heart or the duodenum for several seconds. Besides being a more fruitful mode of diagnosis, it spares the patient a long-time exposure to X-rays.

In its present commercial form the image amplifier is associated with the spot-film device. This is a mechanism by which, when the fluoroscopist sees something he wishes to record, a film holder is advanced quickly by a system of levers to that area and the exposure made.

The image amplifier also became, last year, a traveler. One of the first commercial models made the trip to Copenhagen, Denmark for demonstration before the International Congress of Radiologists—the most important convention of this branch of the medical profession.

It also has taken to the highway. A large trailer—such as is used for mobile tuberculosis survey units—has been fitted with an image amplifier and is on tour to the major cities of the United States to give doctors a first-hand chance to see what it can do. Taking X-ray apparatus to the doctor or clinic for study is in itself a new idea.

Medical men are coming forward with jobs for the image amplifier not contemplated in the earlier stages of its development. It appears that it may have an important function in connection with polio epidemics. One group of doctors are using it to observe the state of a polio-patient's lungs to determine whether he should be given the assistance of the iron lung—a step that, when it becomes necessary, must be taken quickly.

Vacuum Ultra-Violet Spectrometer.—Solar rays found only in the outer atmosphere are reproduced artifically by a new device which will simplify scientific exploration in a little-known region of fundamental research. The device, developed by engineers of the General Electric Company, records the reaction of various crystals to vacuum ultra-violet radiation, the more powerful cousin of ordinary ultra-violet rays that cause sunburn.

A characteristic peculiar to vacuum ultra-violet light is its rapid absorption by air, with the result that the rays never penetrate the earth's layer of atmosphere.

The company's General Engineering Laboratory developed the device as an improvement over existing bulky and complex apparatus.

G–E scientists say that although it produces radiation found only at altitudes of 150 to 200 miles, the new research tool is not intended as a means of studying the problems of space travel.

They say the extensive basic research made possible by the device may lead instead to such developments as an improved phosphor for television picture tubes, better fluorescent lamps, new electronic devices, and a better device for measuring the exposure of human beings to the damaging radiation from atomic bomb blasts.

Although not generally available at this time, this equipment, called a vacuum ultra-violet spectrometer, was designed for maximum flexibility of use. Possible applications include the making of optical absorption measurements on crystals, liquids and gases,

photoconductivity measurements, and for determination of the excitation spectra of phosphors. It also may be used for studies with ordinary ultraviolet light.

Source of the vacuum ultra-violet radiation is a low voltage, high-current hydrogen discharge lamp. The equipment is mounted on a movable table weighing only about 600 lb. and measuring approximately four by five feet. It operates on 110-volt electrical current.

New Use for Cranberries.—A new method for purifying crude ursolic acid, a drug obtained from cranberries, has been developed by University of Wisconsin pharmaceutical researchers who have also converted the acid into a compound which may be useful in prolonging the effect of penicillin injections. The Wisconsin researchers who developed the method are Prof. Lloyd M. Parks and Betty Y. T. Wu.

The substance known as ursolic acid is found in the shiny skins of a number of fruits, among them apples and cranberries, and has possible use in the formulation of emulsions in drug and cosmetic manufacture.

The Wisconsin researchers also produced an amino derivative of ursolic acid which they believe might prove useful in lengthening the effects of penicillin injected into body muscles. A problem always encountered in administering penicillin is that the penicillin is absorbed too rapidly by the body. A drug named procaine is often injected with penicillin to slow absorption, and Parks and Wu believe that the amino derivative might be equally useful for prolonging the activity of penicillin in the body.

The research at Wisconsin was supported by the National Cranberry Association which now has a pilot plant for the experimental production of ursolic acid in operation at Hanson, Mass.

Dental Technician Develops Artificial Mouth.—The brain-child of a Navy enlisted dental technician won second prize when it was placed in competition with other exhibits at the convention of the American Dental Association in Cleveland. The exhibit displayed a machine designed to simulate cavity-producing conditions in the human mouth, originally conceived by Dentalman Barnard A. Yocke during his work in the Dental Research Facility of the Naval Medical Center, Bethesda, Md. Yocke had become intrigued by the idea of being able to watch a cavity as it develops in a tooth and, on his own, developed a crude machine for simulating caries conditions from spare parts and discarded equipment. Naval consultants took notice and gave his work official sanction and expert guidance. The result is a "mouth" (with teeth) that simulates breathing, living pulp chambers, a flow of saliva, body temperature and chewing motions. It hasn't learned to talk—yet.

New Underwater Acoustic Lab in Operation. — The Naval Ordnance Laboratory recently announced that a floating facility for studying long-range, underwater, sound-pickup devices is now in operation on a Patuxent River reservoir north of the Capital, replacing a previous one on nearby Lake Barcroft. Testing is performed from a covered barge with an enclosed well through which experimental devices can be lowered to the bottom of the reservoir. The facility is equipped to work with frequencies ranging from sub-audio to high ultrasonic and will be used for testing sound projectors, hydrophones and transducers, as well as for conducting basic research in the general field of underwater sound.

Fibre-Glass Burnishing Wheel.—A two per cent reduction in the unit cost of Micarta plastic helmet liners has resulted through the use of an improved burnishing wheel now being utilized in the Micarta Division of Westinghouse Electric Corporation.

The new wheel, which is 1 in. thick and 12 in. in diameter, is constructed of alternating layers of fibre glass cloth impregnated with a resin. It is a material similar to that used in the armed forces famed "shrap jackets" for stopping bullets and shell fragments.

Mr. C. B. Dick, manager of Westinghouse Micarta Division, explained that this wheel has proven particularly successful in the finishing of blunt, irregularly contoured edges of army helmet liners. Over half a million of these liners were finished with little or no attention to the burnishing wheels.

In the past, excess flash was removed from the edges of plastic pieces by means of abrasive covered belts. The final finishing operation was performed on short-lived cotton wheels on which alternate coats of cement and abrasive had been applied. Those wheels required constant attention and frequent redressing because of a rapid loss of this coating.

Now this two-stage, costly, and time-consuming operation has been shortened to a one-stage, simplified procedure with the introduction of a glass-laminated burnishing wheel. Grooves are cut in the thick edge of the wheel with a carbide-tipped tool and it is in these grooves that the actual grinding operation is done.

The abrasive action is due to a combination of the glass and high speed. The fact that end, edge, and cut fibres are intermittently coming in contact with the work gives a combination action of prompt removal of flash and polishing in one operation.

The grinding and cutting characteristics of this wheel are further demonstrated by its abrasive action on an ordinary machinists file. No distortion of the wheel groove occurs even after the square corners of the file have been ground down.

Use for Atomic Reactor Heat Waste.—The General Electric Company has disclosed that large quantities of waste heat developed in atomic reactors at the Hanford plutonium-producing plant at Richland, Washington, shortly will be used for the first time in this country for heating buildings.

According to Francis K. McCune, G–E atomic boss, this practical industrial use of the excess heat will be achieved by indirectly using water carrying away reactor heat to warm the air drawn into certain Hanford plant structures now under construction.

The system was designed by engineers of General Electric Company, which operates Hanford for the AEC and the Charles T. Main firm of Architect-Engineers. Actually, several buildings will be heated by one system, but about half the heat will go into the main structure housing a production reactor now under construction.

Enough heat to take care of the needs of a thousand average-sized homes will be extracted from the reactor cooling water and transferred to the air going through plant air conditioners.

The initial investment for the heat recovery system is estimated at about $614,000, and an annual operating cost, excluding repairs, at about $2,200. The estimated fuel saving of $59,000 a year would pay off in 7.5 years the $444,000 of the initial cost of the heat recovery system that would be additional to the cost of a conventional steam system.

As explained by the designing engineers, here is how the system will work.

Water from the Columbia River is pumped through the great Hanford reactors where plutonium is made by the transmission of uranium. The water absorbs immense quantities of heat generated by the process; on its trip through the reactor, dissolved mineral matter in the water becomes slightly radio-active.

Because of this factor, the water from the reactor will be run through a heat exchanger to warm up the water in a secondary piped circuit that will relay the heat to an air conditioning system in the building, while the radio-active water is carried away. This activity quickly dies away by natural decay, but the concentration of large volumes would create a radio-activity hazard to the plant areas before being harmlessly dissipated in the main stream of the Columbia.

To prevent the movement of contaminated dust particles from the reactor to working areas, a constant flow of air is maintained across the reactors to the outside.

Consequently, no air is reheated and recirculated in the building. Instead, a large volume of air per minute is pumped from the outside, cleaned, heated and poured into the building.

To pipe the water direct from the reactor through coils in the floor or walls to warm the building by radiant heat would be impractical because of this large volume of fresh air contantly moving in.

The heating system itself is simple in both concept and in execution. It consists of three major elements connected by an appropriate system of piping. First, the primary exchanger, which transfers heat from the effluent stream to an intermediate fluid—second, the secondary exchangers, which transfer heat from this intermediate fluid to the air—and third, a by-pass exchanger, which is arranged to provide adequate heating when the reactor process is shutdown.

The by-pass exchanger is supplied with steam from a central station plant when called on to perform. The primary purpose of the stream plant is to supply emergency electrical energy to the plant in the event of a power outage. No extra capacity for heating is considered in installing the steam generators.

A network of piping connects the various process buildings with the primary heat exchanger. This piping is all sub-surface and is not insulated, as the primary heat source is both abundant enough and at sufficiently low temperature so that losses to the ground will be considered negligible.

This heat recovery system will answer the question of scores of engineers throughout the nation who have hoped for some economic use of this large energy source. Since September 1944, the huge reactor process plants at Hanford have been pouring vast quantities of waste heat into the Columbia. This heat, a necessary by-product of the nuclear reactions involved in converting uranium to fissionable plutonium, has been of great interest to engineers, who have looked forward to its industrial application.

New High-Speed Facsimile Equipment Developed.—Top military communications experts were recently given an opportunity to study a new high-speed facsimile system which may be used to transmit instantaneously almost any kind of document over long distances. The new equipment, fundamentals of which were conceived by the Navy's Bureau of Ships, has been in the making for three years and is still in the developmental stage. The system makes use of cathode ray

tubes for both transmission and reception. The image received is composed of electrostatically charged "dust" (or dye powder) which clings selectively to a rotating drum sensitized by a moving spot of light. The image is transferred to paper on contact and is "fixed" by baking under infra-red lamps. Copy can be transmitted at the rate of ten 9 × 12-in. pages per minute, with no lengthy processing required of operators at either end of the system. Research and construction were carried out under contracts with the Radio Corporation of America and the Haloid Company.

Pictures of Polio Virus.—Authenticated pictures of an isolated poliomyelitis virus were shown for the first time before the eleventh annual meeting of the Electron Microscope Society of America.

The presentation was made by Dr. A. R. Taylor, research virologist at Parke, Davis & Company, Detroit, in a paper entitled, "Electron Microscopy of Poliomyelitis Virus (Mahoney Tissue Culture Strain)."

The pictures—some magnified 200,000 to 300,000 times—showed the poliomyelitis virus to be a very small spherical particle about 30 millimicrons in diameter. One millimicron is equal to 1/25,000,000 of an inch. The unique pictures thus confirmed previous estimates that the polio virus is one of the smallest of all viruses.

Dr. Taylor explained to the scientists that the Parke-Davis work was done in connection with the development of a polio vaccine and "indicates the feasibility of preparing concentrated virus preparations necessary for the production of a vaccine."

It has been known for sometime that Parke-Davis is working on the development of a polio vaccine. But the company has said only that a vaccine won't be ready for general use until after further development and extensive clinical tests, which may require a year or so more.

Before the electron microscope pictures could be made, a highly potent tissue culture fluid had to be produced, Dr. Taylor pointed out. This involved extremely complicated laboratory procedures, carried out by a virology team which included Dr. F. D. Stimpert, director of microbiological research; Dr. I. W. McLean, Jr., assistant director of microbiological research; and Dr. Arnold E. Hook, research virologist.

Dr. Taylor explained, "A great deal of fundamental information regarding the relationships of the biological, physical and chemical properties of the polio virus is sorely needed in order to adequately control the growth of the virus, and the development and processing of even a crude type of vaccine. Now that we have tissue culture methods, we can study these relationships more effectively."

He and his associates—including Miss Mary Jane McCormick, co-author of the paper—are studying the three known types of polio virus.

Precipitron® Features Push-Button Wash Cycle.—A new low-cost electronic air-cleaning unit, the PX Precipitron®, is available from Westinghouse Electric Corporation. Combining 90 per cent cleaning efficiency with a simple push-button method of spray-washing, the new unit cuts maintenance costs and eliminates inside-the-duct washing of cells.

Accumulated dirt on the electronic collector-plate surfaces of the unit is flushed away by penetrating hot water from needle-spray nozzles attached to a motorized header. After moving slowly across the unit's face, the header automatically reverses and returns to the start position. The same travel-

ing header applies an adhesive oil to the collector-plate surfaces when washing is completed. The oil binds collected dirt together and acts as a solvent for the wash operation.

Height and width of the PX Precipitron® unit depend on the cleaning requirements of a proposed system. Power packs and control units are located outside the duct.

Typical installations will be found in industrial plants, office buildings, department stores, food processing plants, and hospitals.

New 50-Amp., Heavy-Duty Feed-Thru Capacitor.—A new series of high-current, heavy-duty feed-thru capacitors for use in suppressing r-f. interference has just been announced by the Sprague Electric Co., North Adams, Mass. Originally developed for 115/250 volt a-c. lines in mobile military power units, these 50-amp. Thru-Pass* Capacitors are now available for general use.

The entire shell of these interference suppression capacitors is threaded except for two straddle milled flats. When mounted in a flatted circular hole in a chassis or bulkhead wall, they will not rotate and loosen under vibration. There is always a noise-leak-proof path encircling the feed-thru conductor so that the theoretical effectiveness of these capacitors is realized in actual practice.

Designated Type 112P, these heavy-duty Thru-Pass* Capacitors are an outgrowth of Sprague's highly successful r-f. noise suppression capacitors introduced for general applications a year ago. Like them, this new series features glass-to-metal solder seal terminals for protection against atmospheric conditions from −55° C. to +85° C. . . . exclusive Vitamin Q* impregnation for maximum insulation resistance and minimum capacitance change with temperature . . . 250 volts a-c. working voltage.

* Trademark of the Sprague Electric Company.

Germanium Photocell.—A General Electric Research Laboratory scientist has designed a bean-size germanium photocell so versatile it can regulate domestic or industrial heating devices or seek out industrial targets for guided missles. The new electric eye, only slightly thicker than a pencil lead and but three-eighths of an inch long, is more sensitive to light than vacuum photocells a hundred times larger.

Dr. W. C. Dunlap, Jr., scientist who designed the cell, said its acute sensitivity to infrared radiation makes it ideally suited to the regulation and monitoring of various flames and heating devices.

The unit, still in the developmental stage, has a comparatively large power output that should make it extremely useful in many industrial applications.

Because of its power output, the new germanium device could be used for counting, sorting and measuring applications. It could, for example, be employed to reject underfilled containers moving past it on a production line. This magic eye also could serve in military and civilian roles as a substitute for a Geiger counter and similar atomic radiation detection devices. When nuclear particles penetrate a germanium wafer inside the cell, a current is produced. This current, when slightly amplified, can set off light and sound warning apparatus.

The new G–E development is extremely simple in design. It consists of a metal cartridge housing a glass lens and a germanium wafer, in contact with a metallic button of indium. A whisker links the wafer to an outside lead. The device is called a "p-n junction photocell."